A Level

Mathematics

Session: 2010 June
Type: Mark scheme
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GCE

Mathematics

Advanced Subsidiary GCE **4721**

Core Mathematics 1

Mark Scheme for June 2010

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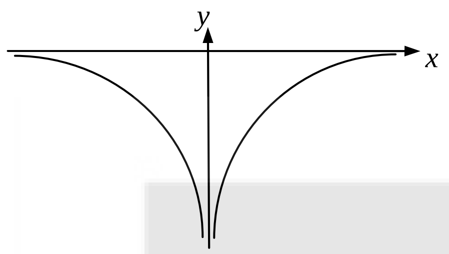
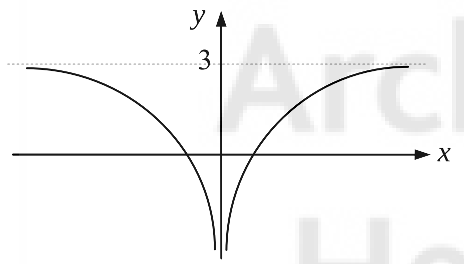
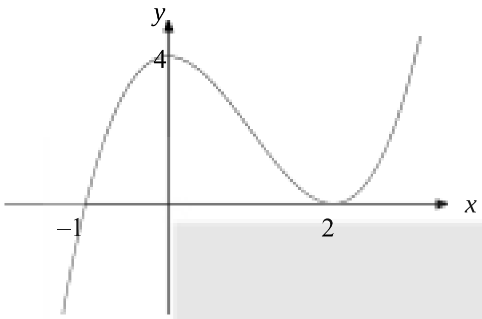
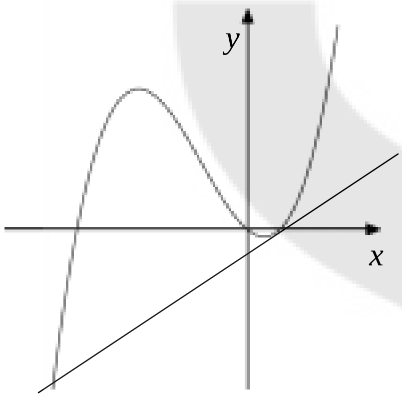
1 (i)	1	**B1**	**1**	
(ii)	$\frac{1}{3}$	**M1**		$\frac{1}{9^{\frac{1}{2}}}$ or $\frac{1}{\sqrt{9}}$ soi
		A1	**2** $\boxed{3}$	cao
2 (i)		**B1***		Reasonably correct curve for $y=-\frac{1}{x^2}$ in 3rd and 4th quadrants only
		B1 dep*	**2**	Very good curves in curve for $y=-\frac{1}{x^2}$ in 3rd and 4th quadrants **SC** If 0, very good single curve in either 3rd or 4th quadrant and nothing in other three quadrants. **B1**
(ii)		**M1**		Translation of their $y=-\frac{1}{x^2}$ vertically
		A1	**2**	Reasonably correct curve, horizontal asymptote soi at $y = 3$
(iii)	$y=-\frac{2}{x^2}$	**B1**	**1** $\boxed{5}$	
3 (i)	$\frac{12(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$	**M1**		Multiply numerator and denom by $3-\sqrt{5}$
	$=\frac{12(3-\sqrt{5})}{9-5}$	**A1**		$(3+\sqrt{5})(3-\sqrt{5})=9-5$
	$=9-3\sqrt{5}$	**A1**	**3**	
(ii)	$3\sqrt{2}-\sqrt{2}$	**M1**		Attempt to express $\sqrt{18}$ as $k\sqrt{2}$
	$=2\sqrt{2}$	**A1**	**2** $\boxed{5}$	

4 (i)	$(x^2-4x+4)(x+1)$	M1		Attempt to multiply a 3 term quadratic by a linear factor or to expand all 3 brackets with an appropriate number of terms (including an x^3 term)
		A1		Expansion with at most 1 incorrect term
	$=x^3-3x^2+4$	A1	3	Correct, simplified answer
(ii)		B1		+ve cubic with 2 or 3 roots
		B1		Intercept of curve labelled (0, 4) or indicated on y-axis
		B1	3 $\boxed{6}$	(–1, 0) and turning point at (2, 0) labelled or indicated on x-axis and no other x intercepts
5	$k=x^2$ $4k^2+3k-1=0$	M1*		Use a substitution to obtain a quadratic or factorise into 2 brackets each containing x^2
	$(4k-1)(k+1)=0$	M1 dep		Correct method to solve a quadratic
	$k=\frac{1}{4}$ (or $k=-1$)	A1		
	$x=\pm\frac{1}{2}$	M1		Attempt to square root to obtain x
		A1	5 $\boxed{5}$	$\pm\frac{1}{2}$ and no other values
6	$y=2x+6x^{-\frac{1}{2}}$	M1		Attempt to differentiate
	$\frac{dy}{dx}=2-3x^{-\frac{3}{2}}$	A1		$kx^{-\frac{3}{2}}$
		A1		Completely correct expression (no +c)
	When $x=4$, gradient $=2-\frac{3}{\sqrt{4^3}}$	M1		Correct evaluation of either $4^{-\frac{3}{2}}$ or $4^{-\frac{1}{2}}$
	$=\frac{13}{8}$	A1	5 $\boxed{5}$	
7	$2(6-2y)^2+y^2=57$	M1*		substitute for x/y or attempt to get an equation in 1 variable only
		A1		correct unsimplified expression
	$2(36-24y+4y^2)+y^2=57$ $9y^2-48y+15=0$	A1		obtain correct 3 term quadratic
	$3y^2-16y+5=0$ $(3y-1)(y-5)=0$	M1 dep		correct method to solve 3 term quadratic
	$y=\frac{1}{3}$ or $y=5$	A1		
	$x=\frac{16}{3}$ or $x=-4$	A1	6 $\boxed{6}$	**SC** If A0 A0, one correct pair of values, spotted or from correct factorisation **www B1**

8 (i)	$2(x^2 + \frac{5}{2}x)$			
	$= 2\left[\left(x+\frac{5}{4}\right)^2 - \frac{25}{16}\right]$	**B1**		$\left(x+\frac{5}{4}\right)^2$
		M1		$q = -2p^2$
	$= 2\left(x+\frac{5}{4}\right)^2 - \frac{25}{8}$	**A1**	**3**	$q = -\frac{25}{8}$ c.w.o.
(ii)	$\left(-\frac{5}{4}, -\frac{25}{8}\right)$	**B1√** **B1√**	**2**	
(iii)	$x = -\frac{5}{4}$	**B1**	**1**	
(iv)	$x(2x+5) > 0$	**M1**		Correct method to find roots
		A1		0, $-\frac{5}{2}$ seen
	$x < -\frac{5}{2}$, $x > 0$	**M1**		Correct method to solve quadratic inequality.
		A1	**4** $\boxed{10}$	(not wrapped, strict inequalities, no 'and')
9 (i)	$\frac{4+p}{2} = -1$, $\frac{5+q}{2} = 3$	**M1**		Correct method (may be implied by one correct coordinate)
	$p = -6$ $q = 1$	**A1** **A1**	**3**	
(ii)	$r^2 = (4-{}^{-}1)^2 + (5-3)^2$	**M1**		Use of $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ for either radius or diameter
	$r = \sqrt{29}$	**A1**	**2**	
(iii)	$(x+1)^2 + (y-3)^2 = 29$	**M1**		$(x+1)^2$ and $(y-3)^2$ seen
		M1		$(x \pm 1)^2 + (y \pm 3)^2 =$ their r^2
	$x^2 + y^2 + 2x - 6y - 19 = 0$	**A1**	**3**	Correct equation in correct form
(iv)	gradient of radius $= \frac{3-5}{-1-4}$	**M1**		uses $\frac{y_2-y_1}{x_2-x_1}$
	$= \frac{2}{5}$	**A1**		oe
	gradient of tangent $= -\frac{5}{2}$	**B1√**		oe
	$y - 5 = -\frac{5}{2}(x-4)$	**M1**		correct equation of straight line through (4, 5), any non-zero gradient
	$y = -\frac{5}{2}x + 15$	**A1**	**5** $\boxed{13}$	oe 3 term equation e.g. $5x + 2y = 30$

10(i)	$\frac{dy}{dx} = 6x^2 + 10x - 4$	B1 B1		1 term correct Completely correct (no +c)
	$6x^2 + 10x - 4 = 0$ $2(3x^2 + 5x - 2) = 0$	M1*		Sets their $\frac{dy}{dx} = 0$
	$(3x-1)(x+2) = 0$	M1 dep*		Correct method to solve quadratic
	$x = \frac{1}{3}$ or $x = -2$ $y = -\frac{19}{27}$ or $y = 12$	A1 A1	6	**SC** If A0 A0, one correct pair of values, spotted or from correct factorisation **www B1**
(ii)	$-2 < x < \frac{1}{3}$	M1 A1	2	Any inequality (or inequalities) involving both their x values from part (i) Allow ≤ and ≥
(iii)	When $x = \frac{1}{2}$, $6x^2 + 10x - 4 = \frac{5}{2}$	M1		Substitute $x = \frac{1}{2}$ into their $\frac{dy}{dx}$
	and $2x^3 + 5x^2 - 4x = -\frac{1}{2}$	B1		Correct y coordinate
	$y + \frac{1}{2} = \frac{5}{2}\left(x - \frac{1}{2}\right)$	M1		Correct equation of straight line using their values. Must use their $\frac{dy}{dx}$ value not e.g. the negative reciprocal
	$10x - 4y - 7 = 0$	A1	4	Shows rearrangement to given equation **CWO** throughout for A1
(iv)		B1 B1	2 $\boxed{14}$	Sketch of a cubic with a tangent which meets it at 2 points only +ve cubic with max/min points and line with +ve gradient as tangent to the curve to the right of the min **SC1** **B1** Convincing algebra to show that the cubic $8x^3 + 20x^2 - 26x + 7 = 0$ factorises into $(2x - 1)(2x - 1)(x + 7)$ **B1** Correct argument to say there are 2 distinct roots **SC2 B1** Recognising y = 2.5x -7/4 is tangent from part (iii) **B1** As second B1 on main scheme

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GCE

Mathematics

Advanced Subsidiary GCE **4722**

Core Mathematics 2

Mark Scheme for June 2010

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1 (i)	$f(2) = 8 + 4a - 2a - 14$ $2a - 6 = 0$ $a = 3$	**M1***		Attempt f(2) or equiv, including inspection / long division / coefficient matching
		M1d*		Equate attempt at f(2), or attempt at remainder, to 0 and attempt to solve
		A1	**3**	Obtain $a = 3$
(ii)	$f(-1) = -1 + 3 + 3 - 14$ $= -9$	**M1**		Attempt f(-1) or equiv, including inspection / long division / coefficient matching
		A1 ft	**2**	Obtain -9 (or $2a - 15$, following their a)
			$\boxed{5}$	
2 (i)	$\text{area} \approx \frac{1}{2} \times 3 \times \left(\sqrt[3]{8} + 2\left(\sqrt[3]{11} + \sqrt[3]{14}\right) + \sqrt[3]{17}\right)$	**B1**		State or imply at least 3 of the 4 correct y-coords , and no others
		M1		Use correct trapezium rule, any h, to find area between $x = 1$ and $x = 10$
	≈ 20.8	**M1**		Correct h (soi) for their y-values – must be at equal intervals
		A1	**4**	Obtain 20.8 (allow 20.7)
(ii)	use more strips / narrower strips	**B1**	**1**	Any mention of increasing n or decreasing h
			$\boxed{5}$	
3 (i)	$(1 + ½x)^{10} = 1 + 5x + 11.25x^2 + 15x^3$	**B1**		Obtain $1 + 5x$
		M1		Attempt at least the third (or fourth) term of the binomial expansion, including coeffs
		A1		Obtain $11.25x^2$
		A1		Obtain $15x^3$
			4	
(ii)	coeff of $x^3 = (3 \times 15) + (4 \times 11.25) + (2 \times 5)$ $= 100$	**M1**		Attempt at least one relevant term, with or without powers of x
		A1 ft		Obtain correct (unsimplified) terms (not necessarily summed) – either coefficients or still with powers of x involved
		A1	**3**	Obtain 100
			$\boxed{7}$	

4 (i)	$u_1 = 6, u_2 = 11, u_3 = 16$	**B1**	**1**	State 6, 11, 16
(ii)	$S_{40} = {}^{40}/_2\,(2 \text{ x } 6 + 39 \text{ x } 5)$ $= 4140$	**M1**		Show intention to sum the first 40 terms of a sequence
		M1		Attempt sum of their AP from (i), with $n = 40$, a = their u_1 and d = their $u_2 - u_1$
		A1	**3**	Obtain 4140
(iii)	$w_3 = 56$ $5p + 1 = 56$ or $6 + (p - 1) \text{ x } 5 = 56$ $p = 11$	**B1**		State or imply $w_3 = 56$
		M1		Attempt to solve $u_p = k$
		A1	**3**	Obtain $p = 11$
			$\boxed{7}$	
5 (i)	$\dfrac{\sin\theta}{8} = \dfrac{\sin 65}{11}$	**M1**		Attempt use of correct sine rule
	$\theta = 41.2^{\circ}$	**A1**	**2**	Obtain 41.2°, or better
(ii) a	$180 - (2 \text{ x } 65) = 50^{\circ}$ or $65 \text{ x } {}^{\pi}/_{180} = 1.134$ $50 \text{ x } {}^{\pi}/_{180} = 0.873$ **A.G.** $\pi - (2 \text{ x } 1.134) = 0.873$	**M1**		Use conversion factor of ${}^{\pi}/_{180}$
		A1	**2**	Show 0.873 radians convincingly (**AG)**
(ii) b	area sector = ½ x 8^2 x 0.873 = 27.9 area triangle = ½ x 8^2 x sin 0.873 = 24.5 area segment = 27.9 – 24.5 = 3.41	**M1**		Attempt area of sector, using (½) $r^2\theta$
		M1		Attempt area of triangle using (½) $r^2 \sin\theta$
		M1		Subtract area of triangle from area of sector
		A1	**4**	Obtain 3.41or 3.42
			$\boxed{8}$	

6 a	$\int_3^5 (x^2+4x)\,dx = \left[\frac{1}{3}x^3 + 2x^2\right]_3^5$	**M1**		Attempt integration
	$= (^{125}/_3 + 50) - (9 + 18)$	**A1**		Obtain $^1/_3x^3 + 2x^2$
	$= 64\ ^2/_3$	**M1**		Use limits $x = 3, 5$ – correct order & subtraction
		A1	**4**	Obtain $64\ ^2/_3$ or any exact equiv
b	$\int (2 - 6\sqrt{y})\,dy = 2y - 4y^{\frac{3}{2}} + c$	**B1**		State $2y$
		M1		Obtain $ky^{\frac{3}{2}}$
		A1	**3**	Obtain $-4y^{\frac{3}{2}}$ (condone absence of $+ c$)
c	$\int_1^\infty 8x^{-3}\,dx = \left[\frac{-4}{x^2}\right]_1^\infty$	**B1**		State or imply $\frac{1}{x^3} = x^{-3}$
	$= (0) - (-4)$	**M1**		Attempt integration of kx^n
	$= 4$	**A1**		Obtain correct $-4x^{-2}$ $(+c)$
		A1 ft	**4**	Obtain 4 (or $-k$ following their kx^{-2})
			$\boxed{11}$	
7 (i)	$\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x}$	**M1**		Use either $\sin^2 x + \cos^2 x = 1$, or $\tan x = {}^{\sin x}/_{\cos x}$
	$= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$			
	$= \tan^2 x - 1$	**A1**	**2**	Use other identity to obtain given answer convincingly.
(ii)	$\tan^2 x - 1 = 5 - \tan x$ $\tan^2 x + \tan x - 6 = 0$ $(\tan x - 2)(\tan x + 3) = 0$ $\tan x = 2$, $\tan x = -3$ $x = 63.4^\circ, 243^\circ$ $\quad x = 108^\circ, 288^\circ$	**B1**		State correct equation
		M1		Attempt to solve three term quadratic in $\tan x$
		A1		Obtain 2 and -3 as roots of their quadratic
		M1		Attempt to solve $\tan x = k$ (at least one root)
		A1ft		Obtain at least 2 correct roots
		A1	**6**	Obtain all 4 correct roots
			$\boxed{8}$	

8 a	$\log 5^{3w-1} = \log 4^{250}$	**M1***		Introduce logarithms throughout
	$(3w - 1)\log 5 = 250 \log 4$	**M1***		Use $\log a^b = b \log a$ at least once
	$3w - 1 = \frac{250 \log 4}{\log 5}$	**A1**		Obtain $(3w - 1)\log 5 = 250 \log 4$ or equiv
	$w = 72.1$	**M1d***		Attempt solution of linear equation
		A1	**5**	Obtain 72.1, or better
b	$\log_x \frac{5y+1}{3} = 4$	**M1**		Use $\log a - \log b = \log {}^a/_b$ or equiv
	$\frac{5y+1}{3} = x^4$	**M1**		Use $f(y) = x^4$ as inverse of $\log_x f(y) = 4$
	$5y + 1 = 3x^4$	**M1**		Attempt to make y the subject of $f(y) = x^4$
	$y = \frac{3x^4 - 1}{5}$	**A1**	**4**	Obtain $y = \frac{3x^4 - 1}{5}$, or equiv
			$\boxed{9}$	
9 (i)	$ar = a + d$, $ar^3 = a + 2d$	**M1**		Attempt to link terms of AP and GP, implicitly or explicitly.
	$2ar - ar^3 = a$ $ar^3 - 2ar + a = 0$ $r^3 - 2r + 1 = 0$ **A.G.**	**M1**		Attempt to eliminate d, implicitly or explicitly, to show given equation.
		A1	**3**	Show $r^3 - 2r + 1 = 0$ convincingly
(ii)	$f(r) = (r - 1)(r^2 + r - 1)$	**B1**		Identify $(r - 1)$ as factor or $r = 1$ as root
		M1*		Attempt to find quadratic factor
	$r = \frac{-1 \pm \sqrt{5}}{2}$	**A1**		Obtain $r^2 + r - 1$
	Hence $r = \frac{-1+\sqrt{5}}{2}$	**M1d***		Attempt to solve quadratic
		A1	**5**	Obtain $r = \frac{-1+\sqrt{5}}{2}$ only
(iii)	$\frac{a}{1-r} = 3 + \sqrt{5}$	**M1**		Equate S_∞ to $3 + \sqrt{5}$
	$a = (\frac{3}{2} - \frac{\sqrt{5}}{2})(3 + \sqrt{5})$	**A1**		Obtain $\frac{a}{1 - \left(\frac{-1+\sqrt{5}}{2}\right)} = 3 + \sqrt{5}$
	$a = {}^9/_2 - {}^5/_2$ $a = 2$	**M1**		Attempt to find a
		A1	**4**	Obtain $a = 2$
			$\boxed{12}$	

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Mathematics

Advanced GCE **4723**

Core Mathematics 3

Mark Scheme for June 2010

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1	(i)	Attempt use of product rule	M1	producing … + … form
		Obtain $3x^2e^{2x}+2x^3e^{2x}$	A1 **2**	or equiv
	(ii)	Attempt use of chain rule to produce $\dfrac{kx}{3+2x^2}$ form	M1	any constant k
		Obtain $\dfrac{4x}{3+2x^2}$	A1 **2**	
	(iii)	Attempt use of quotient rule	M1	or equiv; condone u/v confusions
		Obtain $\dfrac{2x+1-2x}{(2x+1)^2}$ or $(2x+1)^{-1}-2x(2x+1)^{-2}$	A1 **2**	or (unsimplified) equiv
		[If $...+c$ included in all three parts and all three parts otherwise correct, award M1A1, M1A1, M1A0; otherwise ignore any inclusion of $...+c$.]		
			$\boxed{6}$	
2	(i)	Obtain one of $\pm\ln(\pm x\pm 4)$	M1	
		Obtain correct equation $y=-\ln(x-4)$	A1 **2**	or equiv; condone use of modulus signs instead of brackets
	(ii)	State, in any order, S, S and T	M1	or equiv such as S^2, T or 2S, T
		State T, then S, then S	A1 **2**	or equiv (note that S, S, T^9 and S, T^3, S are alternative correct answers)
			$\boxed{4}$	
3	(i)	Use $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$	B1	
		Attempt to express equation in terms of $\sin\theta$	M1	using $\cos 2\theta=\pm1\pm2\sin^2\theta$ or equiv
		Obtain or clearly imply $6\sin^2\theta-11\sin\theta-10=0$	A1 **3**	or $-6\sin^2\theta+11\sin\theta+10=0$
	(ii)	Attempt solution to obtain at least one value of $\sin\theta$	M1	should be $s=-\frac{2}{3},\frac{5}{2}$
		Obtain –41.8	A1	allow –42 or greater accuracy
		Obtain –138	A1 **3**	or greater accuracy; and no others between –180 and 180
		[Answer(s) only: award 0 out of 3.]		
			$\boxed{6}$	

4	(i)	Either: Integrate to obtain $k\ln x$	B1		
		Use at least one relevant logarithm property	M1		
		Obtain $k\ln 3 = \ln 81$ and hence $k = 4$	A1	**3**	AG; accurate work required
		Or 1: (where solution involves no use of a logarithm property)			
		Integrate to obtain $k\ln x$	B1		
		Obtain correct explicit expression for k and conclude $k = 4$ with no error seen	B2	**3**	AG; e.g. $k = \frac{\ln 81}{\ln 6 - \ln 2} = 4$
		Or 2: (where solution involves verification of result by initial substitution of 4 for k)			
		Integrate to obtain $4\ln x$	B1		
		Use at least one relevant logarithm property	M1		
		Obtain $\ln 81$ legitimately with no error seen	A1	**3**	AG; accurate work required
	(ii)	State volume involves $\int \pi(\frac{4}{x})^2\,dx$	B1		possibly implied
		Obtain integral of form $k_1 x^{-1}$	M1		any constant k_1 including π or not
		Use correct process for finding volume produced from S	M1		$\int (k_2 2^2 - k_3 y^2)\,dx$, including π or not with correct limits indicated; or equiv
		Obtain $16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$	A1	**4**	or exact equiv
			$\boxed{7}$		
5	(i)	Attempt process for finding both critical values	M1		squaring both sides to obtain 3 terms on each side or considering 2 different linear eqns/inequalities
		Obtain -4	A1		
		Obtain $\frac{2}{3}$	A1		
		Attempt process for solving inequality	M1		table, sketch, …; needs two critical values; implied by plausible answer
		Obtain $-4 \le x \le \frac{2}{3}$	A1	**5**	with $\le$ and not $<$
	(ii)	Use correct process to find value of $\lvert x+2\rvert$ using any value	M1		… whether part of answer to **(i)** or not
		Obtain $2\frac{2}{3}$ or $\frac{8}{3}$	A1	**2**	dependent on 5 marks awarded in part **(i)**
			$\boxed{7}$		

Q	Part	Answer	Marks	Total	Guidance
6	(i)	Attempt calculations involving 1.0 and 1.1	M1		using radians
		Obtain -0.57 and 0.76	A1		or values to 1 dp (rounded or truncated); or equivs (where eqn rearranged)
		Refer to sign change (or equiv for rearranged eqn)	A1	3	AG; following correct work only
	(ii)	Obtain correct first iterate	B1		using value x_1 such that $1.0 \le x_1 \le 1.1$
		Carry out iteration process	M1		obtaining at least 3 iterates in all so far
		Obtain at least 3 correct iterates	A1		showing at least 3 dp
		Obtain 1.05083	A1	4	answer required to exactly 5 d.p.
		[$1 \to 1.047198 \to 1.050571 \to 1.050809 \to 1.050826 \to 1.050827$; $1.05 \to 1.050769 \to 1.050823 \to 1.050827 \to 1.050827$; $1.1 \to 1.054268 \to 1.051070 \to 1.050844 \to 1.050829 \to 1.050827$]			
	(iii)	State or imply $\sec^2 2x = 1 + \tan^2 2x$	B1		
		Relate to earlier equation	M1		by halving or doubling answer to **(ii)** or carrying out equivalent iteration process
		Deduce $2x = 1.05083$ and hence 0.525	A1√	3	following their answer to **(ii)**; or greater accuracy
		[SC: Rearrange to obtain $x = \frac{1}{2}\cos^{-1}(2x+3)^{-\frac{1}{2}}$	B1		
		Use iterative process to obtain 0.525	B1	2	or greater accuracy]
			$\boxed{10}$		
7		Differentiate to obtain $k_1(3x-1)^3$	M1		any constant k_1
		Obtain correct $12(3x-1)^3$	A1		or (unsimplified) equiv
		Substitute 1 to obtain 96	A1		
		Attempt to find x-coordinate of Q	M1		using tangent with $y = 0$ or using gradient
		Obtain $\frac{5}{6}$	A1		or exact equiv
		Integrate to obtain $k_2(3x-1)^5$	M1		any constant k_2
		Obtain correct $\frac{1}{15}(3x-1)^5$	A1		or (unsimplified) equiv
		Use limits $\frac{1}{3}$ and 1 to obtain $\frac{32}{15}$	A1		
		Attempt to find shaded area by correct process	M1		integral – triangle or equiv
		Obtain ($\frac{32}{15} - \frac{1}{2} \times \frac{1}{6} \times 16$ and hence) $\frac{4}{5}$	A1		or equiv
			$\boxed{10}$		
8	(i)	Obtain $R = 3\sqrt{2}$ or $R = \sqrt{18}$ or $R = 4.24$	B1		or equiv
		Attempt to find value of α	M1		condone sin/cos muddles and degrees
		Obtain $\frac{1}{4}\pi$ or 0.785	A1	3	in radians now
	(ii) a	Equate $x - \alpha$ to $\frac{1}{2}\pi$ or attempt solution of $3\cos x + 3\sin x = 0$	M1		condone degrees here
		Obtain $\frac{3}{4}\pi$	A1	2	or $\ldots, -\frac{5}{4}\pi, -\frac{1}{4}\pi, \frac{7}{4}\pi, \ldots$; in radians now
	b	Attempt correct process to find value of $3x - \alpha$	*M1		with attempt at rearranging $\mathrm{T}(3x) = \frac{8}{9}\sqrt{6}$
		Obtain at least one correct exact value of $3x - \alpha$	A1		$\pm\frac{1}{6}\pi, \pm\frac{11}{6}\pi, \ldots$
		Attempt at least one positive value of x	M1		dep *M
		Obtain $\frac{1}{36}\pi$	A1	4	
			$\boxed{9}$		

9	**(i)**	Attempt to find x-coord of staty point or complete square	M1		
		Obtain $(\frac{3}{2},-9)$ or $4(x-\frac{3}{2})^2-9$ or -9	A1		or equiv
		State $\mathrm{f}(x) \geq -9$	A1	**3**	using any notation; with $\geq$
	(ii)	Make one correct (perhaps general) relevant statement	B1		not 1 –1 , f is many-one, … ; maybe implied if attempt is specific to this f
		Conclude with correct evidence related to this f	B1	**2**	AG; (more or less) correct sketch; correct relevant calculations, …
	(iii)	<u>Either</u>: Attempt to find expression for g^{-1}	*M1		or equiv
		Obtain $\frac{1}{a}(x-b)$	A1		or equiv
		Compare $\frac{1}{a}(x-b)$ and $ax+b$	M1		dep *M; by equating either coefficients of x or constant terms (or both); or substituting two non-zero values of x and solving eqns for a
		Obtain at least $-\frac{b}{a}=b$ and hence $a=-1$	A1	**4**	AG; necessary detail required; or equiv
		[SC1: first two steps as above, then substitute $a=-1$: max possible M1A1B1]			
		[SC2: substitute $a=-1$ at start: Attempt to find inverse M1 Obtain $-x+b$ and conclude A1 **2**]			
		<u>Or</u>: State or imply that $y=\mathrm{g}^{-1}(x)$ is reflection of $y=\mathrm{g}(x)$ in line $y=x$	B1		
		State that line unchanged by this reflection is perpendicular to $y=x$	M2		
		Conclude that a is –1	A1	**4**	
	(iv)	State or imply that $\mathrm{gf}(x)=-(4x^2-12x)+b$	B1		
		Attempt use of discriminant or relate to range of f	M1		or equiv
		Obtain $64+16b<0$ or $9+b<5$	A1		or equiv
		Obtain $b<-4$	A1	**4**	
			$\boxed{\textbf{13}}$		

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GCE

Mathematics

Advanced GCE **4724/01**

Core Mathematics 4

Mark Scheme for June 2010

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1	First 2 terms in expansion = $1-5x$	B1	(simp to this, now or later)
	3rd term shown as $\frac{-\frac{5}{3}.-\frac{8}{3}}{2}(3x)^2$	M1	$-\frac{8}{3}$ can be $-\frac{5}{3}-1$ $(3x)^2$ can be $9x^2$ or $3x^2$
	$= +\ 20x^2$	A1	
	4th term shown as $\frac{-\frac{5}{3}.-\frac{8}{3}.-\frac{11}{3}}{2.3}(3x)^3$	M1	$-\frac{11}{3}$ can be $-\frac{5}{3}-2$ $(3x)^3$ can be $27x^3$ or $3x^3$
	$= -\frac{220}{3}x^3$ ISW	A1	Accept $-\frac{440}{6}x^3$ ISW

N.B. If 0, SR B2 to be awarded for $1-\frac{5}{3}x+\frac{20}{9}x^2-\frac{220}{81}x^3$. Do not mark $(1+x)^{-5/3}$ as a MR.

$\boxed{5}$

2	Attempt quotient rule [Show fraction with denom $(1-\sin x)^2$ & num $+/-(1-\sin x) +/- \sin x +/- \cos x +/- \cos x$]	M1	
	Numerator = $(1-\sin x).-\sin x - \cos x.-\cos x$ { Product symbols must be clear or implied by further work }	A1	terms in any order
	Reduce correct numerator to $1-\sin x$	B1	or $-\sin x+\sin^2 x+\cos^2 x$
	Simplify to $\frac{1}{1-\sin x}$ ISW	A1	Accept $-\frac{1}{\sin x-1}$

$\boxed{4}$

3	$\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}$	M1	For correct format
	$A(x-1)(x-2)+B(x-2)+C(x-1)^2 \equiv x^2$	M1	
	$A=-3$	A1	
	$B=-1$	A1	(B1 if cover-up rule used)
	$C=4$	A1	(B1 if cover-up rule used)

[NB1: Partial fractions need not be written out; correct format + correct values sufficient.
NB2: Having obtained B & C by cover-up rule, candidates may substitute into general expression & algebraically manipulate; the M1 & A1 are then available if deserved.]

$\boxed{5}$

<u>These special cases using different formats are the only other ones to be considered</u>		<u>Max</u>
$\frac{A}{x-1}+\frac{Bx+C}{(x-1)^2}+\frac{D}{x-2}$;	M1 M1; A0 for any values of A, B & C, A1 or B1 for $D=4$	3
$\frac{Ax+B}{(x-1)^2}+\frac{C}{x-2}$;	M0 M1; A1 for $A=-3$ <u>and</u> $B=2$, A1 or B1 for $C=4$	3

4		Att by diff to connect dx & du or find $\frac{dx}{du}$ or $\frac{du}{dx}$ (not dx=du)	M1	no accuracy; not 'by parts'
		d$x = 2u$ du or $\frac{du}{dx} = \frac{1}{2}(x+2)^{-\frac{1}{2}}$ AEF	A1	
		Indefinite integral $\rightarrow \int 2(u^2-2)^2 (\frac{u}{u})(du)$	A1	May be implied later
		{If relevant, cancel u/u and} attempt to square out	M1	
		{dep $\int k\mathrm{I}(du)$ where $k=2$ or $\frac{1}{2}$ or 1 and I = $(u^2-2)^2$ or $(2-u^2)^2$ or $(u^2+2)^2$ }		
		Att to change limits if working with f(u) after integration	M1	or re-subst into integral attempt and use -1 & 7
		Indef integ = $\frac{2}{5}u^5 +/- \frac{8}{3}u^3 + 8u$ or $\frac{1}{10}u^5 +/- \frac{2}{3}u^3 + 2u$	A1	or $\frac{1}{5}u^5 +/- \frac{4}{3}u^3 + 4u$
		$\frac{652}{15}$ or $43\frac{7}{15}$ ISW but no '+c'	A1	
			$\boxed{7}$	
5		$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$ s.o.i.	B1	Implied by e.g., $4x\frac{dy}{dx} + y$
		$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1	
		Diff eqn(=0 can be implied)(solve for $\frac{dy}{dx}$ and) put $\frac{dy}{dx} = 0$	M1	
		Produce only $2x+4y=0$ (though AEF acceptable)	*A1	without any error seen
		Eliminate x or y from curve eqn & eqn(s) just produced	M1	
		Produce either $x^2 = 36$ or $y^2 = 9$	dep* A1	Disregard other solutions
		$(\pm 6, \mp 3)$ AEF, as the only answer ISW	dep* A1	Sign aspect must be clear
			$\boxed{7}$	
6	(i)	State/imply scalar product of any two vectors = 0	M1	
		Scalar product of correct two vectors = 4 + 2a –6	A1	($4+2a-6=0 \rightarrow$ M1A1)
		$a=1$	A1 **3**	
	(ii)	**(a)** Attempt to produce at least two relevant equations	M1	e.g. $2t = 3+2s$ …..
		Solve two not containing 'a' for s and t	M1	
		Obtain at least one of $s = -\frac{1}{2}$, $t=1$	A1	
		Substitute in third equation & produce $a = -2$	A1 **4**	
		(b) Method for finding magnitude of any vector	M1	possibly involving 'a'
		Using $\cos\theta = \frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\boldsymbol{b}\rvert}$ for the pair of direction vectors	M1	possibly involving 'a'
		107, 108 (107.548) or 72, 73, 72.4, 72.5 (72.4516) c.a.o.	A1 **3**	1.87, 1.88 (1.87707) or 1.26
			$\boxed{10}$	

7	(i)	Differentiate x as a quotient, $\frac{v\,\mathrm{d}u - u\,\mathrm{d}v}{v^2}$ or $\frac{u\,\mathrm{d}v - v\,\mathrm{d}u}{v^2}$	M1	or product clearly defined	
		$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{(t+1)^2}$ or $\frac{-1}{(t+1)^2}$ or $-(t+1)^{-2}$	A1	WWW $\rightarrow$ 2	
		$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{2}{(t+3)^2}$ or $\frac{-2}{(t+3)^2}$ or $-2(t+3)^{-2}$	B1		
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$	M1	quoted/implied and used	
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2(t+1)^2}{(t+3)^2}$ or $\frac{2(t+3)^{-2}}{(t+1)^{-2}}$ (dep 1st 4 marks)	*A1	ignore ref $t=-1, t=-3$	
		State <u>squares</u> +ve or $(t+1)^2$ & $(t+3)^2$ + ve $\therefore \frac{\mathrm{d}y}{\mathrm{d}x}$ +ve	dep*A1 **6**	or $\left(\frac{t+1}{t+3}\right)^2$ + ve . Ignore ≥ 0	
	(ii)	Attempt to obtain t from either the x or y equation	M1	No accuracy required	
		$t = \frac{2-x}{x-1}$ AEF <u>or</u> $t = \frac{2}{y} - 3$ AEF	A1		
		Substitute in the equation not yet used in this part	M1	or equate the 2 values of t	
		Use correct meth to eliminate ('double-decker') fractions	M1		
		Obtain <u>$2x + y = 2xy + 2$</u> ISW AEF	A1 **5**	but not involving fractions	$\boxed{\mathbf{11}}$
8	(i)	<u>Long division method</u>		<u>Identity method</u>	
		Evidence of division process as far as 1st stage incl sub	M1	$\equiv Q(x-1)+R$	
		(Quotient =) $x-4$	A1	$Q = x-4$	
		(Remainder =) 2 ISW	A1 **3**	$R=2$; N.B. might be B1	
	(ii)	**(a)** Separate variables; $\int \frac{1}{y-5}\,\mathrm{d}y = \int \frac{x^2-5x+6}{x-1}\,\mathrm{d}x$	M1	'$\int$' may be implied later	
		Change $\frac{x^2-5x+6}{x-1}$ into their (Quotient + $\frac{\text{Rem}}{x-1}$)	M1		
		$\ln(y-5)$ =√ (integration of their previous result) (+c)ISW	√A1 **3**	f.t. if using Quot + $\frac{\text{Rem}}{x-1}$	
	(ii)	**(b)** Substitute $y = 7$, $x = 8$ into their eqn containing 'c'	M1	& attempt 'c' (-3.2 , $\ln\frac{2}{49}$)	
		Substitute $x = 6$ and their value of 'c'	M1	& attempt to find y	
		<u>$y = 5.00$ (5.002529)</u> Also $5 + \frac{50}{49}e^{-6}$	A2 **4**	Accept 5, 5.0,	
		Beware: <u>any</u> wrong working anywhere $\rightarrow$ A0 even if answer is one of the acceptable ones.			
			$\boxed{\mathbf{10}}$		

9 (i)	Attempt to multiply out $(x+\cos 2x)^2$	M1	Min of 2 correct terms
	<u>Finding $\int 2x\cos 2x\,dx$</u>		
	Use $u=2x, dv=\cos 2x$	M1	1st stage $f(x)+/-\int g(x)\,dx$
	1st stage $x\sin 2x-\int \sin 2x\,dx$	A1	
	$\therefore \int 2x\cos 2x\,dx = x\sin 2x+\frac{1}{2}\cos 2x$	A1	
	<u>Finding $\int \cos^2 2x\,dx$</u>		
	Change to $k\int +/-1+/-\cos 4x\,dx$	M1	where $k=\frac{1}{2}$, 2 or 1
	Correct version $\frac{1}{2}\int 1+\cos 4x\,dx$	A1	
	$\int \cos 4x\,dx = \frac{1}{4}\sin 4x$	B1	seen anywhere in this part
	Result = $\frac{1}{2}x+\frac{1}{8}\sin 4x$	A1	
	(i) ans $=\frac{1}{3}x^3+x\sin 2x+\frac{1}{2}\cos 2x+\frac{1}{2}x+\frac{1}{8}\sin 4x$ (+ c)	A1 **9**	Fully correct
(ii)	V = $\pi\int_0^{\frac{1}{2}\pi}(x+\cos 2x)^2\ (dx)$	M1	
	Use limits 0 & $\frac{1}{2}\pi$ correctly on their **(i)** answer	M1	
	(**i**) correct value = $\frac{1}{24}\pi^3-\frac{1}{2}+\frac{1}{4}\pi-\frac{1}{2}$	A1	
	Final answer = $\pi\left(\frac{1}{24}\pi^3+\frac{1}{4}\pi-1\right)$	A1 **4**	**c.a.o.** No follow-through
		$\boxed{13}$	

<u>Alternative methods</u>

2 If $y=\frac{\cos x}{1-\sin x}$ is changed into $y(1-\sin x)=\cos x$, award

M1 for clear use of the product rule (though possibly trig differentiation inaccurate)

A1 for $-y\cos x+(1-\sin x)\frac{dy}{dx}=-\sin x$ AEF

B1 for reducing to a fraction with $1-\sin x$ or $-\sin x+\sin^2 x+\cos^2 x$ in the numerator

A1 for correct final answer of $\frac{1}{1-\sin x}$ or $(1-\sin x)^{-1}$

If $y=\frac{\cos x}{1-\sin x}$ is changed into $y=\cos x(1-\sin x)^{-1}$, award

M1 for clear use of the product rule (though possibly trig differentiation inaccurate)

A1 for $\left(\frac{dy}{dx}\right)=\cos^2 x(1-\sin x)^{-2}+(1-\sin x)^{-1}.-\sin x$ AEF

B1 for reducing to a fraction with $1-\sin x$ or $-\sin x+\sin^2 x+\cos^2 x$ in the numerator

A1 for correct final answer of $\dfrac{1}{1-\sin x}$ or $(1-\sin x)^{-1}$

6(ii)(a) If candidates use some long drawn-out method to find 'a' instead of the direct route, allow

M1 as before, for producing the 3 equations

M1 for any satisfactory method which will/does produce 'a', however involved

A2 for $a=-2$

7(ii) Marks for obtaining this Cartesian equation are not available in part (i).

<u>If part (ii) is done first and then part (i) is attempted using the Cartesian equation, award marks as follow:</u>

<u>Method 1 where candidates differentiate implicitly</u>

M1 for attempt at implicit differentiation

A1 for $\frac{dy}{dx}=\frac{2y-2}{1-2x}$ AEF

M1 for substituting parametric values of x and y

A2 for simplifying to $\frac{2(t+1)^2}{(t+3)^2}$

A1 for finish as in original method

<u>Method 2 where candidates manipulate the Cartesian equation to find x = or y=</u>

M1 for attempt to re-arrange so that either $y=f(x)$ or $x=g(y)$

A1 for correct $y=\frac{2-2x}{1-2x}$ AEF or $x=\frac{2-y}{2-2y}$ AEF

M1 for differentiating as a quotient

A2 for obtaining $\frac{dy}{dx}=\frac{2}{(1-2x)^2}$ or $\frac{(2-2y)^2}{2}$

A1 for finish as in original method

8(ii)(b) If definite integrals are used , then

M2 for $[\ ]_y^7=[\ ]_6^8$ or equivalent or M1 for $[\ ]_7^y=[\ ]_6^8$ or equivalent

A2 for 5, 5.0, 5.00 (5.002529) <u>with caveat as in main scheme</u> dep M2

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GCE

Mathematics

Advanced GCE **4725**

Further Pure Mathematics 1

Mark Scheme for June 2010

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Q	Part	Answer	Mark	Total	Guidance
1			B1		Establish result true for n = 1or n = 2
			M1		Add next term to given sum formula
			M1		Attempt to factorise or expand and simplify to correct expression
			A1		Correct expression obtained
			A1	5	Specific statement of induction conclusion
				$\boxed{5}$	
2	(i)	$\begin{pmatrix}-7\end{pmatrix}$	M1		Obtain a single value
			A1	2	Obtain correct answer as a matrix
	(ii)	BA = $\begin{pmatrix}5 & -20\\ 3 & -12\end{pmatrix}$	M1		Obtain a 2 $\times$ 2 matrix
			A1		All elements correct
		$\begin{pmatrix}-7 & -20\\ 11 & -20\end{pmatrix}$	B1		4**C** seen or implied by correct answer
			B1ft	4	Obtain correct answer, ft for a slip in **BA**
				$\boxed{6}$	
3		Either	M1		Express as a sum of 3 terms
			M1		Use standard sum results
		$\frac{2}{3}n(n+1)(2n+1)-2n(n+1)+n$	A1		Correct unsimplified answer
			M1		Attempt to factorise
			A1		Obtain at least factor of n and a quadratic
		$\frac{1}{3}n(2n-1)(2n+1)$	A1	6	Obtain correct answer a.e.f.
		Or			
		$\sum_{r=1}^{2n} r^2 - 4\sum_{r=1}^{n} r^2$	M1		Express as difference of 2 $\sum r^2$ series
			M1		Use standard result
		$\frac{1}{6}\times 2n(2n+1)(4n+1)-4\times\frac{1}{6}n(n+1)(2n+1)$	A1		Correct unsimplified answer
			M1		Attempt to factorise
			A1		Obtain at least factor of n
		$\frac{1}{3}n(2n-1)(2n+1)$	A1		Obtain correct answer
				$\boxed{6}$	

4	**(i)**		5 + 12i	B1B1	Correct real and imaginary parts
			13	B1ft	Correct modulus
			67.4° or 1.18	B1ft **4**	Correct argument
	(ii)			M1	Multiply by conjugate
				A1	Obtain correct numerator
			$-\frac{11}{85}-\frac{27}{85}\text{i}$	A1 **3**	Obtain correct denominator
				$\boxed{7}$	
5	**(a)**		$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	B1B1**2**	Each column correct **SC B2 use correct matrix from MF1** **Can be trig form**
	(b)	**(i)**		B1B1**2**	Stretch, in x-direction sf 5
		(ii)		B1B1**2**	Rotation, 60° clockwise
				$\boxed{6}$	
6	**(i)**	**(a)**		B1B1**2**	Circle centre (3, –4), through origin
		(b)		B1B1**2**	Vertical line, clearly x = 3
	(ii)			B1ft	Inside their circle
				B1ft **2**	And to right of their line, if vertical
				$\boxed{6}$	

7	*Either*	B1B1	State or use correct results
	$\alpha+\beta=-2k \quad \alpha\beta=k$	M1	Attempt to find sum of new roots
		A1	Obtain $4k$
		M1	Attempt to find product of new roots
		A1	Obtain $4k$
		B1ft **7**	Correct quadratic equation a.e.f.
	$y^2-4ky+4k=0$		
	Or		
	$\alpha+\beta=-2k$	B1	State or use correct result
	$\frac{-2k}{\alpha}$	B1	State or imply form of new roots
	$y=\frac{-2k}{x}$	B1	State correct substitution
		M1	Rearrange and substitute for x
		A1	Correct unsimplified equation
		M1	Attempt to clear fractions
	$y^2-4ky+4k=0$	A1	Correct quadratic equation a.e.f.
	Or		
	$-k\pm\sqrt{k^2-k}$	B1	Find roots of original equation
	$\frac{\alpha+\beta}{\alpha}=\frac{2k}{k+\sqrt{k^2-k}},\ \frac{\alpha+\beta}{\beta}=\frac{2k}{k-\sqrt{k^2-k}}$	B1	Express both new roots in terms of k
		M1	Attempt to find sum of new roots
		A1	Obtain $4k$
		M1	Attempt to find product of new roots
	$y^2-4ky+4k=0$	A1	Obtain $4k$
		B1ft	Correct quadratic equation a.e.f.

$\boxed{7}$

8	(i)		M1		Attempt to rationalise denominator or cross multiply
			A1	2	Obtain **given** answer correctly
	(ii)		M1		Express terms as differences using (i)
			M1		Attempt this for at least 1st three terms
			A1		1st three terms all correct
			A1		Last two terms all correct
			M1		Show pairs cancelling
		$\frac{1}{2}(\sqrt{n+2}+\sqrt{n+1}-\sqrt{2}-1)$	A1	6	Obtain correct answer, in terms of n
	(iii)		B1	1	Sensible statement for divergence
				$\boxed{9}$	
9	(i)		M1		Show correct expansion process for 3 x 3
			M1		Correct evaluation of any 2 x 2
		det **A** = $a^2 - a$	A1	3	Obtain correct answer
	(ii)	(a)	M1		Find a pair of inconsistent equations
			A1		State inconsistent or no solutions
		(b)	M1		Find a repeated equation
			A1		State non unique solutions
		(c)	B1		State that det **A** is non-zero or find correct solution
			B1	6	State unique solution
					SC if detA incorrect, can score 2 marks for correct deduction of a unique solution, but only once
				$\boxed{9}$	
10	(i)		M1		Attempt to equate real and imaginary parts
		$x^2 - y^2 = 3 \quad xy = 2$	A1		Obtain both results
			M1		Eliminate to obtain quadratic in x^2 or y^2
			M1		Solve to obtain x or y value
		$z = 2 + \mathrm{i}$	A1	5	Obtain correct answer as a complex no.
	(ii)		B1	1	Obtain **given** answer correctly
	(iii)		M1		Attempt to solve quadratic equation
		$w^3 = 2 \pm 11\mathrm{i}$	A1		Obtain correct answers
			M1		Choose negative sign
			M1		Relate required value to conjugate of (i)
		$w = 2 - \mathrm{i}$	A1	5	Obtain correct answer
				$\boxed{11}$	

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GCE

Mathematics

Advanced GCE **4726**

Further Pure Mathematics 2

Mark Scheme for June 2010

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PO Box 5050
Annesley
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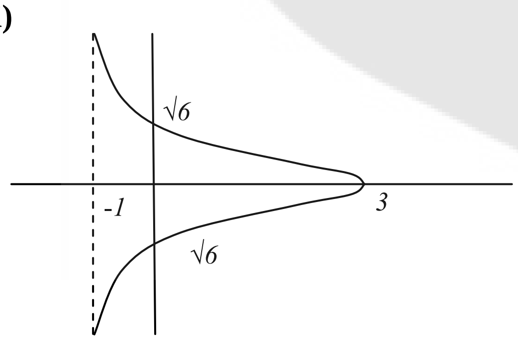
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1	Derive/quote $g'(x) = p/(1+x^2)$	B1	
	Attempt $f'(x)$ as $a/(1+bx^2)$	M1	Allow any a, b=2 or 4
	Use $x = ½$ to set up a solvable equation in p, leading to at least one solution	M1	
	Get $p = {}^5/_4$ only	A1	AEEF
2	Reasonable attempt at e^{2x} $(1+2x+2x^2)$	M1	3 terms of the form $1+2x+ax^2$, $a\neq0$
	Multiply out their expressions to get all terms up to x^2	M1	(3 terms) x (minimum of 2 terms)
	Get $1+3x+4x^2$	A1	cao
	Use binomial, equate coefficients to get 2 solvable equations in a and n	M1	Reasonable attempt at binomial, each term involving a and n (an=3, $a^2n(n-1)/2$=4)
	Reasonable attempt to eliminate a or n	M1	
	Get n=9, a=⅓ cwo	A1	cao
			SC Reasonable $f'(x)$ and $f''(x)$ using product rule (2 terms) M1
			Use their expressions to find $f'(0)$ and $f''(0)$ M1
			Get $1+3x+4x^2$ cao A1
3	Quote/derive correct $dx=2dt/(1+t^2)$	B1	
	Replace all x (not $dx=dt$)	M1	From their expressions
	Get $2/(t-1)^2$ or equivalent	A1	
	Reasonable attempt to integrate their expression	M1	
	Use correct limits in their correct integral	A1√	Must involve √3
	Clearly tidy to √3+1 from cwo	A1	A.G.
4 (i)	Get a = -2	B1	May be quoted (from correct working)
	Get b = 6	B1	May be quoted (from correct working)
	Get c = 1	B1	May be quoted (from correct working)

(ii)



B1 Correct shape in $-1<x\leq3$ only (allow just top or bottom half)

B1 90^0 (at x=3) (must cross x-axis i.e. symmetry)

B1 Asymptote at x= -1 only (allow −1 seen)

B1√ Correct crossing points; $\pm\sqrt{(b/c)}$ from their b,c

5 (i) Reasonable attempt at parts	M1	Leading to second integral
Get $e^x(1-2x)^n - \int e^x.n(1-2x)^{n-1}.-2\,dx$	A1	Or $(1-2x)^{n+1}/(-2(n+1))e^x$ $-\int(1-2x)^{n+1}/(-2(n+1))e^x dx$
Evidence of limits used in integrated part	M1	Should show ± 1
Tidy to A.G.	A1	Allow $I_{n+1} = 2(n+1)I_n - 1$
(ii) Show any one of $I_3=6I_2-1$, $I_2=4I_1-1$, $I_1=2I_0-1$	B1	May be implied
Get $I_0(=e^{½}-1)$ or $I_1(=2e^{½}-3)$	B1	
Substitute their values back for their I_3	M1	Not involving n
Get $48e^{½} - 79$	A1	
6 (i) Reasonable attempt to differentiate $\sinh y = x$ to get dy/dx in terms of y	M1	Allow $\pm\cosh y\ dy/dx = 1$
Replace $\sinh y$ to A.G.	A1	Clearly use $\cosh^2 - \sinh^2 = 1$
		SC Attempt to diff. $y = \ln(x+\sqrt{(x^2+1)})$ using chain rule M1
		Clearly tidy to A.G. A1
(ii) Reasonable attempt at chain rule	M1	To give a product
Get $dy/dx = a\sinh(a\sinh^{-1}x)/\sqrt{(x^2+1)}$	A1	
Reasonable attempt at product/quotient	M1	Must involve sinh and cosh
Get d^2y/dx^2 correctly in some form	A1√	From $dy/dx = k\sinh(a\sinh^{-1}x)/\sqrt{(x^2+1)}$
Substitute in and clearly get A.G.	A1	
		SC Write $\sqrt{(x^2+1)}dy/dx = k\sinh(a\sinh^{-1}x)$ or similar
		Derive the A.G.
7 (i) Get 5.242, 5.239, 5.237	B1√	Any 3(minimum) correct from previous value
Get 5.24	B1	Allow one B1 for 5.24 seen if 2 d.p.used
(ii) Show reasonable staircase for any region	B1	Drawn curve to line
Describe any one of the three cases	B1	
Describe all three cases	B1	
(iii) Reasonable attempt to use log/expo. rules	M1	Allow derivation either way
Clearly get A.G.	A1	
Attempt $f'(x)$ and use at least once in correct N-R formula	M1	
Get answers that lead to 1.31	A1	Minimum of 2 answers; allow truncation/rounding to at least 3 d.p.
(iv) Show $f'(\ln 36) = 0$	B1	
Explain why N-R would not work	B1	Tangent parallel to Ox would not meet Ox again or divide by 0 gives an error

8 (i)	Use correct definition of cosh x	B1	
	Attempt to cube their definition involving e^x and e^{-x} (or e^{2x} and e^x)	M1	Must be 4 terms
	Put their 4 terms into LHS and attempt to simplify	M1	
	Clearly get A.G.	A1	
			SC Allow one B1 for correct derivation from $\cosh 3x = \cosh(2x+x)$
(ii)	Rewrite as $k\cosh 3x = 13$	M1	
	Use ln equivalent on $13/k$	M1	Allow ±ln or $\ln(13/k \pm\sqrt{(13/k)^2 - 1})$ for their k or attempt to set up and solve quadratic via exponentials
	Get $x = (\pm)\ \frac{1}{3}\ln 5$	A1	
	Replace in cosh x for u	M1	
	Use $e^{a\ln b} = b^a$ at least once	M1	
	Get $\frac{1}{2}(5^{\frac{1}{3}} + 5^{-\frac{1}{3}})$	A1	
9 (i)	Attempt integral as $k(2x+1)^{1.5}$	M1	
	Get 9	A1	cao
	Attempt subtraction of areas	M1	Their answer – triangle
	Get 3	A1√	Their answer − 6 (>0)
(ii)	Use $r^2=x^2+y^2$ and $x=r\cos\theta$, $y=r\sin\theta$	B1	
	Eliminate x and y to produce quadratic equation (=0) in r (or $\cos\theta$)	M1	
	Solve their quadratic to get r in terms of θ (or vice versa)	A1√	
	Clearly get A.G.	A1	$r>0$ may be assumed
	Clearly show θ_1(at B)$=\tan^{-1}\frac{3}{4}$ and θ_2 (at A) $=\pi$	B1	
			SC Eliminate y to get r in terms of x only M1 Get $r = x + 1$ A1 SC Start with $r=1/(1-\cos\theta)$ and derive cartesian
(iii)	Use area $= \frac{1}{2}\int r^2\, d\theta$ with correct r	B1	cwo; ignore limits
	Rewrite as $k\operatorname{cosec}^4(\frac{1}{2}\theta)$	M1	Not just quoted
	Equate to their part **(i)** and tidy	M1	To get $\int$ = some constant
	Get 24	A1	A.G.

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RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **4727**

Further Pure Mathematics 3

Mark Scheme for June 2010

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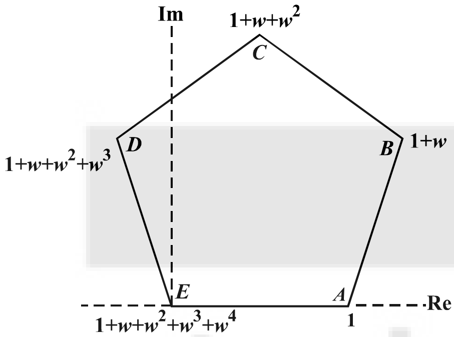
1	Direction of $l_1 = k[7, 0, -10]$ Direction of $l_2 = k[1, 3, -1]$	B1	For both directions
	EITHER $\mathbf{n} = [7, 0, -10] \times [1, 3, -1]$ *OR* $\begin{cases} [x, y, z] \,.\, [7, 0, -10] = 0 \Rightarrow 7x - 10z = 0 \\ [x, y, z] \,.\, [1, 3, -1] = 0 \Rightarrow x + 3y - z = 0 \end{cases}$	M1	For finding vector product of directions of l_1 and l_2 *OR* for using 2 scalar products and obtaining equations
	$\Rightarrow \mathbf{n} = k[10, -1, 7]$	A1	For correct $\mathbf{n}$
	METHOD 1 Vector $(\mathbf{a} - \mathbf{b})$ from l_1 to $l_2 = \pm[4, 6, -10]$ *OR* $\pm[-4, 3, 1]$ *OR* $\pm[3, 3, -9]$ *OR* $\pm[-3, 6, 0]$	B1	For a correct vector
	$d = \dfrac{\lvert(\mathbf{a} - \mathbf{b}) \,.\, \mathbf{n}\rvert}{\lvert\mathbf{n}\rvert} = \dfrac{36}{\sqrt{150}}$	M1*	For finding $(\mathbf{a} - \mathbf{b}) \,.\, \mathbf{n}$
		M1 (*dep)	For $\lvert\mathbf{n}\rvert$ in denominator *OR* for using $\hat{\mathbf{n}}$
	$d = \frac{6}{5}\sqrt{6} \approx 2.94$	A1 **7**	For correct distance **AEF**
	METHOD 2 Planes containing l_1 and l_2 perp. to $\mathbf{n}$	M1*	For finding planes and $p_1 - p_2$ seen
	are $\mathbf{r} \,.\, [10, -1, 7] = p_1 = 70$, $\mathbf{r} \,.\, [10, -1, 7] = p_2 = 34$	B1	For $p_1 = 70k$ and $p_2 = 34k$
	$\Rightarrow d = \dfrac{\lvert 70 - 34\rvert}{\sqrt{150}} = \dfrac{36}{\sqrt{150}} = \frac{6}{5}\sqrt{6} \approx 2.94$	M1 (*dep)	For $\lvert\mathbf{n}\rvert$ in denominator *OR* for using $\hat{\mathbf{n}}$
		A1	For correct distance **AEF**
	METHOD 3 $\mathbf{r}_1 = [7\lambda, 0, 10 - 10\lambda]$ *OR* $[7 + 7\lambda, 0, -10\lambda]$ $\mathbf{r}_2 = [4 + \mu, 6 + 3\mu, -\mu]$ *OR* $[3 + \mu, 3 + 3\mu, 1 - \mu]$	B1	For correct points on l_1 and l_2 using different parameters
	$\begin{array}{rc\|c\|c\|c\|c\|} 7\lambda + 10\alpha - \mu & = & 4 & -3 & 3 & -4 \\ -\alpha - 3\mu & = & 6 & 6 & 3 & 3 \\ -10\lambda + 7\alpha + \mu & = & -10 & 0 & -9 & 1 \end{array}$ $\Rightarrow \alpha = -\frac{6}{25}$	M1*	For setting up 3 linear equations from $\mathbf{r}_1 + \alpha\mathbf{n} = \mathbf{r}_2$ and solving for α
	$\lvert\mathbf{n}\rvert = \sqrt{150}$	M1 (*dep)	For $\lvert\mathbf{n}\rvert$ seen multiplying α
	$\Rightarrow d = \frac{6}{25}\sqrt{150} = \frac{6}{5}\sqrt{6} \approx 2.94$	A1	For correct distance **AEF**
		$\boxed{7}$	

2	(i)	$ar = r^5 a \Rightarrow rar = r^6 a$	M1		Pre-multiply $ar = r^5 a$ by r
		$r^6 = e \Rightarrow rar = a$	A1	2	Use $r^6 = e$ and obtain answer **AG**
	(ii)	METHOD 1			
		For $n = 1$, $rar = a$ *OR* For $n = 0$, $r^0 ar^0 = a$	B1		For stating true for $n = 1$ *OR* for $n = 0$
		Assume $r^k ar^k = a$			
		EITHER Assumption $\Rightarrow$ $r^{k+1} ar^{k+1} = rar = a$	M1		For attempt to prove true for $k + 1$
		OR $r^{k+1} ar^{k+1} = r.r^k ar^k.r = rar = a$			
		OR $r^{k+1} ar^{k+1} = r^k.rar.r^k = r^k ar^k = a$	A1		For obtaining correct form
		Hence true for all $n \in \mathbb{Z}^+$	A1	4	For statement of induction conclusion
		METHOD 2			
		$r^2 ar^2 = r.rar.r = rar = a$, similarly for $r^3 ar^3 = a$	M1		For attempt to prove for $n = 2, 3$
		$r^4 ar^4 = r.r^3 ar^3.r = rar = a$, similarly for $r^5 ar^5 = a$	A1		For proving true for $n = 2, 3, 4, 5$
		$r^6 ar^6 = eae = a$	B1		For showing true for $n = 6$
		For $n > 6$, $r^n = r^{n \bmod 6}$, hence true for all $n \in \mathbb{Z}^+$	A1		For using n mod 6 and correct conclusion
		METHOD 3			
		$r^n ar^n = r^{n-1}.rar.r^{n-1}$	M1		Starting from n, for attempt to prove true for $n-1$
		OR $r^n ar^n = r^n.r^5 a.r^{n-1} = r^{n+5} ar^{n-1}$			
		$= r^{n-1} ar^{n-1}$	A1		For proving true for $n-1$
		$= r^{n-2} ar^{n-2} = \ldots$	A1		For continuation from $n-2$ downwards
		$= rar = a$	B1		For final use of $rar = a$
					SR can be done in reverse
		METHOD 4			
		$ar = r^5 a \Rightarrow ar^2 = r^5 ar = r^{10} a$ etc.	M1		For attempt to derive $ar^n = r^{5n} a$
		$\Rightarrow ar^n = r^{5n} a$	A1		For correct equation **SR** may be stated without proof
		$\Rightarrow r^n ar^n = r^{6n} a$	B1		For pre-multiplication by r^n
		$= ea = a$	A1		For obtaining a ($r^6 = e$ may be implied)
				$\boxed{6}$	

3					Allow $\text{cis}\frac{k}{5}\pi$ and $e^{\frac{k}{5}\pi i}$ throughout
	(i)	$w^2 = \cos\frac{4}{5}\pi + i\sin\frac{4}{5}\pi$	B1		For correct value
		$w^3 = \cos\frac{6}{5}\pi + i\sin\frac{6}{5}\pi$	B1		For correct value
		$w^* = \cos\frac{2}{5}\pi - i\sin\frac{2}{5}\pi$	B1		For w^* seen or implied
		$= \cos\frac{8}{5}\pi + i\sin\frac{8}{5}\pi$	B1	4	For correct value **SR** For exponential form with i missing, award B0 first time, allow others
	(ii)		B1*		For $1+w$ in approximately correct position
			B1 (*dep)		For $AB \approx BC \approx CD$
			B1 (*dep)		For BC, CD equally inclined to Im axis
			B1	4	For E at the origin
					Allow points joined by arcs, or not joined Labels not essential
	(iii)	$z^5 - 1 = 0$ *OR* $z^5 + z^4 + z^3 + z^2 + z = 0$	B1	1	For correct equation **AEF** (in any variable) Allow factorised forms using w, exp or trig
				$\boxed{9}$	
4	(i)	$y = xz \Rightarrow \frac{dy}{dx} = z + x\frac{dz}{dx}$	B1		For correct differentiation of substitution
		$\Rightarrow xz + x^2\frac{dz}{dx} - xz = x\cos z \Rightarrow x\frac{dz}{dx} = \cos z$	M1 A1		For substituting into DE For DE in variables separable form
		$\Rightarrow \int \sec z \, dz = \int \frac{1}{x} dx$	M1		For attempt at integration to ln form on LHS
		$\Rightarrow \ln(\sec z + \tan z) = \ln kx$ *OR* $\ln\tan\left(\frac{1}{2}z + \frac{1}{4}\pi\right) = \ln kx$	A1		For correct integration (k not required here)
		$\Rightarrow \sec\left(\frac{y}{x}\right) + \tan\left(\frac{y}{x}\right) = kx$ *OR* $\tan\left(\frac{y}{2x} + \frac{1}{4}\pi\right) = kx$	A1	6	For correct solution **AEF** including $\text{RHS} = e^{(\ln x)+c}$
	(ii)	$(4, \pi) \Rightarrow \sec\frac{1}{4}\pi + \tan\frac{1}{4}\pi = 4k$ *OR* $\tan\left(\frac{1}{8}\pi + \frac{1}{4}\pi\right) = 4k$	M1		For substituting $(4, \pi)$ into their solution (with k)
		$\Rightarrow \sec\left(\frac{y}{x}\right) + \tan\left(\frac{y}{x}\right) = \frac{1}{4}\left(1+\sqrt{2}\right)x$ *OR* $\tan\left(\frac{y}{2x} + \frac{1}{4}\pi\right) = \left(\frac{1}{4}\tan\frac{3}{8}\pi\right)x$ *or* $\frac{1}{4}\left(1+\sqrt{2}\right)x$	A1	2	For correct solution **AEF** Allow decimal equivalent $0.60355x$ Allow $e^{\ln x}$ for x
				$\boxed{8}$	

Question		Answer	Marks		Guidance
5	(i)	$C+\mathrm{i}S=1+\frac{1}{2}\mathrm{e}^{\mathrm{i}\theta}+\frac{1}{4}\mathrm{e}^{2\mathrm{i}\theta}+\frac{1}{8}\mathrm{e}^{3\mathrm{i}\theta}+\ldots$	M1		For using $\cos n\theta+\mathrm{i}\sin n\theta=\mathrm{e}^{\mathrm{i}n\theta}$ at least once for $n\geqslant 2$
			A1		For correct series
		$=\dfrac{1}{1-\frac{1}{2}\mathrm{e}^{\mathrm{i}\theta}}=\dfrac{2}{2-\mathrm{e}^{\mathrm{i}\theta}}$	M1		For using sum of infinite GP
			A1	4	For correct expression **AG** **SR** For omission of 1st stage award up to M0 A0 M1 A1 **OEW**
	(ii)	$C+\mathrm{i}S=\dfrac{2\left(2-\mathrm{e}^{-\mathrm{i}\theta}\right)}{\left(2-\mathrm{e}^{\mathrm{i}\theta}\right)\left(2-\mathrm{e}^{-\mathrm{i}\theta}\right)}$	M1		For multiplying top and bottom by complex conjugate
		$=\dfrac{4-2\mathrm{e}^{-\mathrm{i}\theta}}{4-2\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)+1}=\dfrac{4-2\cos\theta+2\mathrm{i}\sin\theta}{4-4\cos\theta+1}$	M1		For reverting to cosθ and sinθ and equating Re *OR* Im parts
		$\Rightarrow C=\dfrac{4-2\cos\theta}{5-4\cos\theta},\quad S=\dfrac{2\sin\theta}{5-4\cos\theta}$	A1		For correct expression for C **AG**
			A1	4	For correct expression for S
				$\boxed{8}$	
6	(i)	Aux. equation $m^2+2m+17(=0)$	M1		For attempting to solve correct auxiliary equation
		$\Rightarrow m=-1\pm 4\mathrm{i}$	A1		For correct roots
		CF $(y=)\ \mathrm{e}^{-x}\left(A\cos 4x+B\sin 4x\right)$	A1√		For correct CF (allow $A\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}(4x+\varepsilon)$) (trig terms required, not $\mathrm{e}^{\pm 4\mathrm{i}x}$) f.t. from their m with 2 arbitrary constants
		PI $(y=)\ px+q\ \Rightarrow\ 2p+17(px+q)=17x+36$	M1		For stating and substituting PI of correct form
		$\Rightarrow\ p=1$	A1		For correct value of p
		and $q=2$	A1		For correct value of q
		GS $y=\mathrm{e}^{-x}\left(A\cos 4x+B\sin 4x\right)+x+2$	B1√	7	For GS. f.t. from their CF+PI with 2 arbitrary constants in CF and none in PI. Requires $\boxed{y=}$.
	(ii)	$x\gg 0\Rightarrow \mathrm{e}^{-x}\to 0$ *OR* very small	B1		For correct statement. Allow graph
		$\Rightarrow y=x+2$ approximately	B1√	2	For correct equation Allow ≈ , → and in words Allow relevant f.t. from linear part of GS
				$\boxed{9}$	

7	(i)	$(1, 3, 5)$ and $(5, 2, 5) \Rightarrow \pm[4, -1, 0]$ in Π	M1		For finding a vector in Π
		$\mathbf{n} = [2, -2, 3] \times [4, -1, 0] = k[1, 4, 2]$	M1		For finding vector product of direction vectors of l and a line in Π
			A1		For correct $\mathbf{n}$
		$\Rightarrow \mathbf{r}\,.[1, 4, 2] = 23$	A1	4	For correct equation. Allow multiples
	(ii)	METHOD 1			
		Perpendicular to Π through $(-7, -3, 0)$ meets Π	M1		For using perpendicular from point on l to Π Award mark for $k\mathbf{n}$ used
		where $(-7+k)+4(-3+4k)+2(2k)=23$	M1		For substituting parametric line coords into Π
		$\Rightarrow k=2 \quad \Rightarrow d = 2\sqrt{1^2+4^2+2^2} = 2\sqrt{21} \approx 9.165$	M1		For normalising the $\mathbf{n}$ used in this part
			A1	4	For correct distance **AEF**
		METHOD 2			
		Π is $x+4y+2z=23$	M1		For attempt to use formula for perpendicular distance
		$\Rightarrow d = \dfrac{\lvert(-7)+4(-3)+2(0)-23\rvert}{\sqrt{1^2+4^2+2^2}} = 2\sqrt{21} \approx 9.165$	M1		For substituting a point on l into plane equation
			M1		For normalising the $\mathbf{n}$ used in this part
			A1		For correct distance **AEF**
		METHOD 3			
		$\mathbf{m} = [1, 3, 5] - [-7, -3, 0] = (\pm)[8, 6, 5]$ *OR* $= [5, 2, 5] - [-7, -3, 0] = (\pm)[12, 5, 5]$	M1		For finding a vector from l to Π
		$\Rightarrow d = \dfrac{\mathbf{m}\,.[1, 4, 2]}{\sqrt{1^2+4^2+2^2}} = \dfrac{42}{\sqrt{21}} = 2\sqrt{21} \approx 9.165$	M1		For finding $\mathbf{m}\,.\,\mathbf{n}$
			M1		For normalising the $\mathbf{n}$ used in this part
			A1		For correct distance **AEF**
		METHOD 4			As Method 1, using parametric form of Π
		$[-7, -3, 0] + k[1, 4, 2] = [1, 3, 5] + s[2, -2, 3] + t[4, -1, 0]$	M1		For using perpendicular from point on l to Π Award mark for $k\mathbf{n}$ used
		$\left.\begin{array}{l} k-2s-4t = 8 \\ 4k+2s+t = 6 \\ 2k-3s = 5 \end{array}\right\} \Rightarrow k=2 \quad \left(s=-\frac{1}{3},\ t=-\frac{4}{3}\right)$	M1		For setting up and solving 3 equations
		$\Rightarrow d = 2\sqrt{1^2+4^2+2^2} = 2\sqrt{21} \approx 9.165$	M1		For normalising the $\mathbf{n}$ used in this part
			A1		For correct distance **AEF**
		METHOD 5			
		$d_1 = \dfrac{23}{\sqrt{1^2+4^2+2^2}} = \dfrac{23}{\sqrt{21}}$	M1		For attempt to find distance from O to Π *OR* from O to parallel plane containing l
		$d_2 = \dfrac{[-7, -3, 0]\,.[1, 4, 2]}{\sqrt{1^2+4^2+2^2}} = \dfrac{-19}{\sqrt{21}}$	M1		For normalising the $\mathbf{n}$ used in this part
		$\Rightarrow d_1 - d_2 = d = \dfrac{23-(-19)}{\sqrt{21}} = 2\sqrt{21} \approx 9.165$	M1		For finding $d_1 - d_2$
			A1		For correct distance **AEF**
	(iii)	$(-7, -3, 0) + k(1, 4, 2)$	M1		State or imply coordinates of a point on the reflected line
		Use $k=4$	M1		State or imply 2 × distance from **(ii)** Allow $k = \pm 4$ *OR* $\pm 4\sqrt{21}$ f.t. from **(ii)**
		$\mathbf{b} = [2, -2, 3]$	B1		For stating correct direction
		$\mathbf{a} = [-3, 13, 8]$	A1	4	For correct point seen in equation $\mathbf{r} = \mathbf{a} + t\mathbf{b}$
		$\mathbf{r} = [-3, 13, 8] + t[2, -2, 3]$			**AEF** in this form

12

Question	Answer	Marks		Guidance
8 (i)	$\{A, D\}$ *OR* $\{A, E\}$ *OR* $\{A, F\}$	B1	**1**	For stating any one subgroup
(ii)	A is the identity	B1		For identifying A as the identity
	5 is not a factor of 6 *OR* elements can be only of order 1, 2, 3, 6	B1	**2**	For reference to factors of 6
(iii)		M1		For finding BE and EB AND using $\omega^3=1$
	$BE=\begin{pmatrix}0&1\\1&0\end{pmatrix}=D$, $\quad EB=\begin{pmatrix}0&\omega\\\omega^2&0\end{pmatrix}=F$	A1		For correct BE (D *or* matrix)
		A1		For correct EB (F *or* matrix)
	D *or* $\begin{pmatrix}0&1\\1&0\end{pmatrix}$, F *or* $\begin{pmatrix}0&\omega\\\omega^2&0\end{pmatrix}\in M$ $\Rightarrow$ closure property satisfied	A1	**4**	For justifying closure
(iv)	$B^{-1}=\frac{1}{1}\begin{pmatrix}\omega^2&0\\0&\omega\end{pmatrix}=C$	M1		For correct method of finding either inverse
		A1		For correct $B^{-1}=C$ Allow $\begin{pmatrix}\omega^2&0\\0&\omega\end{pmatrix}$
	$E^{-1}=\frac{1}{-1}\begin{pmatrix}0&-\omega^2\\-\omega&0\end{pmatrix}=E$	A1	**3**	For correct $E^{-1}=E$ Allow $\begin{pmatrix}0&\omega^2\\\omega&0\end{pmatrix}$
(v)	METHOD 1			
	M is not commutative e.g. from $BE\neq EB$ in part **(iii)**	B1		For justification of M being not commutative
	N is commutative (as × mod 9 is commutative)	B1		For statement that N is commutative
	$\Rightarrow M$ and N not isomorphic	B1#	**3**	For correct conclusion
	METHOD 2			
	Elements of M have orders 1, 3, 3, 2, 2, 2	B1*		For all orders of one group correct
	Elements of N have orders 1, 6, 3, 2, 3, 6	B1 (*dep)		For sufficient orders of the other group correct
	Different orders *OR* self-inverse elements $\Rightarrow M$ and N not isomorphic	B1#		For correct conclusion **SR** Award up to B1 B1 B1 if the self-inverse elements are sufficiently well identified for the groups to be non-isomorphic
	METHOD 3			
	M has no generator since there is no element of order 6	B1		For all orders of M shown correctly
	N has 2 *OR* 5 as a generator	B1		For stating that N has generator 2 *OR* 5
	$\Rightarrow M$ and N not isomorphic	B1#		For correct conclusion
	METHOD 4 (tables below)	B1*		For stating correctly all 6 squared elements of one group
		B1 (*dep)		For stating correctly sufficient squared elements of the other group
	$\Rightarrow M$ and N not isomorphic	B1#		For correct conclusion
				# In all Methods, the last B1 is dependent on at least one preceding B1
		13		

METHOD 4

M	A	B	C	D	E	F
A	A	B	C	D	E	F
B	B	C	A	F	D	E
C	C	A	B	E	F	D
D	D	E	F	A	B	C
E	E	F	D	C	A	B
F	F	D	E	B	C	A

N	1	2	4	8	7	5
1	1	2	4	8	7	5
2	2	4	8	7	5	1
4	4	8	7	5	1	2
8	8	7	5	1	2	4
7	7	5	1	2	4	8
5	5	1	2	4	8	7

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1 Hills Road
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CB1 2EU

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Facsimile: 01223 552627
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GCE

Mathematics

Advanced Subsidiary GCE **4728**

Mechanics 1

Mark Scheme for June 2010

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OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1 i	$t = 5/1.2$ $t = 4.17$ s	M1 A1 [2]	$5=1.2t$ or $0=5-1.2t$ 4 1/6 s, 4.166 or better, 4.16 recurring.
ii	$s = (-5)^2/2\text{x}1.2$ $s = 10.4$ m *OR (using(i))* $s = 5\text{x}4.17 – 1.2\text{x}4.17^2/2$ $s = 10.4$ m *OR (using(i))* $s = (5 (+ 0))/2 \text{ x } 4.17$ $s = 10.4$ m	M1 A1 [2] M1 A1 M1 A1	$s = 5^2/2\text{x}1.2$ or $5^2 = 2\text{x}1.2s$ or $0 = 5^2 - 2\text{x}1.2s$ Accept 10 5/12, but not 10 Time must be > 0. Accept |t| from (i) Award if |-4.17| used.
iii	Fr = 3x1.2 R = 3x9.8 $\mu = (3\text{x})1.2/(3\text{x})9.8$ $\mu = 0.122$ *OR* R = 3x9.8 Mass x acceleration = +/- 3x1.2 $+/-\mu \text{ x}29.4 = +/- 3\text{x}1.2$ $\mu = 0.122$	B1 B1 M1 A1 [4] B1 B1 M1 A1	Accept 3.6, +/- Accept 3g, +/- Ratio of 2 positive numerical force terms Not 0.12 Accept 3g, +/- Either both positive or both negative.

2 i	+/-(0.4x3 - 0.6x1.5) +/-(0.4x0.1 + 0.6v) (0.4x3 - 0.6x1.5) = +/-(0.4x0.1 + 0.6v) speed $\|v\| = 0.433$ ms^{-1} *OR* +/- (0.4x3 – 0.4x0.1) = +/- 1.16 (0.6v + 0.6x1.5) = 0.6v + 0.9 1.16 = +/-(0.6v + 0.9) speed $\|v\| = 0.433$ ms^{-1}	B1 B1 M1 A1 [4] B1 B1 M1 A1	+/- 0.3 Nb the terms have same signs Equating their total mom before & after Accept 13/30 or 0.43 recurring, but not 0.43 Momentum change of P Momentum change of Q Equating momentum changes 0.26/0.6 = v
ii	+/-(0.4x0.1 - 0.6v) (0.4x3 - 0.6x1.5) = +/-(0.6v - 0.4x0.1) v = 0.567 PQ = 0.1x3 +0.567x3 PQ = 2 m *OR* +/- 0.4x3 + 0.4x0.1 and +/- 0.6v + 0.6x1.5 1.24 = +/- 0.6v + 0.9 v = 0.567 etc	B1 M1 A1 M1 A1 [5] B1 M1 A1	Nb the terms have different signs Must use +/- same before momentum as in (i) May be implied, or in any format (0.1 + 0.567)x3 Accept 2.00(1), 2.0, 2.00 Both must be correct Equating change in momentum May be implied, or in any format

3 i	H = +/-(9 - 5cos60) H = 6.5 N AG	M1 A1 [2]	+/-(9 + 5cos120)
ii	V = +/-(12 - 5sin60) V = 7.67 N	M1 A1 [2]	+/-(12 + 5cos150) Accept 7.666 or better, or 7.6 recurring
iii	$R^2 = 6.5^2 + 7.67^2$ R = 10.1 N tan A = 6.5/7.67 or 7.67/6.5 A = 40(.3) or 49.7 Bearing = 320°	M1 A1 M1 A1 A1 [5]	Uses Pythagoras on forces V(ii) and 6.5 10.053.. Uses trigonometry in relevant triangle May be implied by final answer As this is not a final answer, exact accuracy is not an issue Or better

4 i	$3.2 - 0.2t^2 = 0$ t = 4 s	M1 A1 [2]	Puts 0 for v and attempts to solve QE Accept dual solution +/-4
ii	a = -2x0.2t a = -0.4x4 a = -1.6 ms^{-2}	M1* D*M1 A1 [3]	Differentiates v Substitutes +ve t(i) in derivative of v Negative only
iii	 s= $3.2t - 0.2t^3/3$ (+c) t = 0, s = 0 so c = 0 s(4) = 3.2x4 -0.2x4^3/3 s = 8.53 m	M1* A1 B1 D*M1 A1 [5]	Integrates v, not multiplication by t Or correct use of limits 0 and 4 Accept without/loss of c 8 8/15 Accept with/without c

5 i	+/-3x20/2 30 m	M1 A1 [2]	Use area of <u>scalene</u> triangle(s). Not suvat. Accept -30
ii	 (t+4)x3/2 = 30 or 3t/2 = 30 - 4x3 t = 16 or t = 12 T = 76	M1 A1 A1 A1 [4]	Equates <u>scalene</u> trapezium area to distance (i) [(T-60)+4]x3/2 =30, award A2
iii	T(accn) = 3/0.4 (=7.5 s) decn = 3/([76-60] - 4 - 7.5) decn = (+/-) 2/3 ms^{-2} *OR* S(accn) = 3^2/(2x0.4) (= 11.25 m) decn = 3^2 / [2x(30 – 3x4 – 11.25)] decn = (+/-) 2/3 ms^{-2}	B1 M1 A1 [3] B1 M1 A1	 Or 3 = decn x ([76-60] - 4 - 7.5) (+/-) 0.667 or better - accept 0.6 recurring (+/-) 0.667 or better - accept 0.6 recurring

6 i a	T - 0.85g sin30 = 0.85a 0.55g - T = 0.55a a = 1.225/1.4 a = 0.875 T = 4.91	B1 B1 M1 A1 A1 [5]	Either equation correct Both eqns correct and consistent ‘a’ direction Solves 2 sim eqn 4.908 or better – has to be positive
b	F = 2Tcos30 F = 8.5(02..)	M1 A1ft [2]	Or Pythagoras or cosine rule cv(4.91)x√3
ii	 $v^2 = 1.3^2$ + 2x0.875x1.5 (=4.315) a = +/-gsin30 0 = 4.315 -2x4.9s (s = 0.44…) S = 1.94	M1 A1ft B1 M1 A1 A1 [6]	Uses $v^2 = u^2$ + 2a(1.5), u non-zero, a from (i) v = 2.077....(v^2 =1.69+3xcv(0.875)) a = +/-4.9 Uses 0^2= u^2+/-2as, with a not g or (i), u not1.3 May be implied – need not be 3sf

7 i	Fr = 4 + 5sin60 Fr = 8.33 R = 12 - 5cos60 R = 9.5 μ = (4 + 5sin60)/(12 - 5cos60) μ = 0.877	M1 A1 M1 A1 M1 A1 [6]	All 4 + component 5 (4 + 4.333(01)) May be implied +/-(All 12 – component 5 (12 – 2.5)) May be implied, +ve from correct work Friction/Reaction, Fr>4, R<12, both positive
ii	Upper block μ = 5sin60/(9-5cos60) (=4.3/6.5) μ = 0.666	 M1 A1 [2]	 (Component 5)/(9-component 5)
iii	Upper mass = 9/g (9/g)a = 5sin60 - 0.1(9 - 5cos60) a = 4.01 Lower mass Tractive force = 4 + 0.1(9-5cos60) (= 4.65) Max Friction = 0.877(3+(9-5cos60) (= 8.33) Tractive force < Max Friction a = 0 *OR for Lower Mass* ma = 4+0.1(9-5cos60)-0.877(3+9-5cos60) -ve a caused by friction impossible, hence a = 0	B1 M1 A1 M1 A1 A1 [6] M1 A1 A1	0.918(36..) N2L 0.918(36..)a= 4.33(01..) – 0.1x6.5 where friction = 0.1x(9-component 5) Compares TF (tractive force) and max friction N2L with 3 force terms:

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RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **4729**

Mechanics 2

Mark Scheme for June 2010

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NOTTINGHAM
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1	$v^2 = 2 \times 9.8 \times 10$	M1	Using $v^2 = u^2 + 2as$ with $u = 0$
	$v = 14\ \text{m s}^{-1}$	A1	
	speed = $\sqrt{(7^2 + 14^2)}$	M1	Method to find speed using their "v"
	15.7 or $7\sqrt{5}\ \text{m s}^{-1}$	A1	
	$\tan^{-1}(14/7)$ or $\tan^{-1}(7/14)$	M1	Method to find angle using their "v"
	63.4° to the horizontal	A1 **6**	26.6° to vertical
			6
2 (i)	$(6\sin \Pi/2) \div (\Pi/2)$	M1	Use of correct formula
	3.82	A1 **2**	**AG**
(ii)	8đ = 3(6-3.82) + 5x9.82	M1	Method to find centre of mass
	or 8x = ±{3(-3.82) + 5x3.82}	A1	
	đ = 6.95 or 6.96 or x = +/-0.955	A1	
	$\tan\theta = 0.96/6$	M1	Attempt to find the required angle
	$\theta = 9°$	A1 **5**	
			7
3 (i)	D = 128 000/80 (= 1600)	B1	
	$k(80)^2 = 128\ 000/80$	M1	Driving force = resistance
		A1	
	k = ¼	A1	
	R = 900 N FT	B1 **5**	FT on their k (R = 3600k)
(ii)	D = 128 000 / 60 (= 2133⅓)	B1	
	2000 x 9.8 x sin2°	B1	
	6400/3-900-2000 x 9.8 x sin2° = 2000a	M1	4 terms required
	$a = 0.275\ \text{m s}^{-2}$	A1 **4**	
			9
4 (i)	4Tcos20° = 5 x g x 2.5	M1	Using moments; allow sin/cos mix
		A1	Allow with omission of g
	T = 32.6 N	A1 **3**	
(ii)	X = Tsin20°	M1	allow sin/cos mix
	X = 11.1 FT	A1	FT their T
	Y + Tcos20° = 5 x g or 2.5Y = 1.5 x Tcos20 or 4Y = 1.5 x 5g	M1	
	Y = 18.4 FT	A1	FT their T, but not from omission of g
	$R = \sqrt{(X^2 + Y^2)}$ or $\tan^{-1}(Y/X)$ or $\tan^{-1}(X/Y)$	M1	X ≠ 0, Y ≠ 0
	R = 21.5 N	A1	
	θ = 58.8° above the horizontal	A1 **7**	or 31.2° to left of vertical
			10

5 (i)	$T\cos45° + R\sin45° = mg$	*M1 A1	3 terms
	$T\sin45° - R\cos45° = ml\sin45°\omega^2$	*M1 A1	3 terms; $a = r\,\omega^2$
	$2T = \sqrt{2}mg + ml\omega^2$ $T = m/2(\sqrt{2}g + l\omega^2)$	Dep*M1 A1 **6**	Method to eliminate R **AG** www
(ii)	$R = 0$ $2R = \sqrt{2}mg - ml\omega^2$ or $T\cos45° = mg$ or $T = ml\omega^2$	B1 B1	may be implied
	Solve to find ω	M1	
	$\omega = 4.16$ rad s^{-1}	A1 **4**	**10**

6 (i)	$2mu = 2mv + 3mv$	M1 A1	Conservation of momentum
	$v = 2/5\ u$	A1 **3**	Must be v =
(ii)	$e = (3v - v)\ /\ u$ $e = 4/5$	M1 A1 **2**	Using restitution **AG**
(iii)	Initial K.E. $= 9mv^2 / 2 = 18mu^2 /25$ Final K.E. $= 9mv^2 / 8 = 9mu^2 /50$ $\frac{1}{2} m (V)^2$ = Final K.E. $V = 3\ u\ /5$	B1 FT B1 FT M1 A1 **4**	FT on their v from (i) FT on their v from (i) **AG**
(iv)	$4mu / 5 - 3mu /5 = 2mx + my$ $u / 5 = 2x + y$ $e = 4/5 = (y - x)\ /\ u$ $4u = 5y - 5x$ solving 2 relevant equations $x = -u/5\ y = 3u/5$ $y = 3u/5$ away from wall (x) + towards wall (y)	M1 A1 FT M1 FT A1 M1 A1 A1 A1 **8**	Conservation of momentum FT on their v from (i); aef Using restitution FT on their v from (i); aef both **17**

7 (i)	R = 0.2 x 9.8 x cos30° (= 1.70) F = 0.1 x 9.8 x cos30° (= 0.849) FT ½ x 0.2 x 11^2 - ½ x 0.2 v^2 = 0.2 x 9.8 x 5sin30 + 5 x 0.849 v = 5.44 m s^{-1}	B1 B1 M1 A1 A1 A1	 6	 FT on their R, but not R =0.2g Use of conservation of energy **AG**
Or last 4 marks of (i)	F + 0.2gsin30 = ±0.2a a = ±9.1 $v^2 = 11^2$ + 2 x a x 5 v = 5.44 m s^{-1}	M1 A1 M1 A1		Use of N2L, 3 terms Complete method to find v
(ii)	t = 5cos30°/5.44cos30° t = 0.919 s u = 5.44sin30° (= 2.72) s = 2.72 x 0.919 – 4.9 x 0.919^2 s = -1.6 (or better) Ht drop to *C* = 5sin30° = 2.5 m Ball does not hit the roof	M1 A1 B1 M1 A1 B1 A1	 7	time to lateral position over *C* Ht dropped **13**
Or first 5 marks of (ii)	y = xtanθ – $gx^2 \sec^2\theta/2V^2$ substitute values V = 5.44 θ = 30° x = 5cos30° y = 2.5 – 9.8x25x3/4x4/3 / (2x5.44^2) y = -1.6 (or better)	B1 M1 A1 A1 A1		 all 3 correct
OR (ii)	u = 5.44sin30° (= 2.72) -2.5 = 5.44sin30t – $4.9t^2$ t = 1.04 x = 5.44cos30 x 1.04 = 4.9 (or better) Horizontal distance from B to C = 5cos30 = 4.3(or better) Ball does not hit the roof	B1 M1 A1 A1 A1 B1 A1	 7	 aef time to position level with *AC*
OR (ii)	y = xtanθ – $gx^2 \sec^2\theta/2V^2$ substitute values -2.5 = 0.577x – $0.221x^2$ Attempt to solve quadratic for x x = 4.9 (or better) Horizontal distance from B to C = 5cos30 = 4.3 (or better) Ball does not hit the roof	B1 M1 A1 M1 A1 B1 A1	 7	 aef
OR (ii)	u = 5.44sin30° = 2.72 -2.5 = 5.44sin30t – $4.9t^2$ t = 1.0 (or better) T = 5cos30°/5.44cos30° T = 0.92 (or better) Ball does not hit the roof	B1 M1 A1 A1 M1 A1 A1	 7	 aef time to position level with *AC* time to lateral position over *C*

OR (ii)	Attempt at equation of trajectory $y = 0.577x - 0.221x^2$ $y = -0.577x$ Solving their quadratic and linear equations to get at least x or y x = 5.2 (or better) or y = -3.0 (or better) Horizontal distance from B to C = 5cos30 = 4.3 (or better) Or Ht drop to *C* = 5sin30° = 2.5 Ball does not hit the roof	M1 A1 B1 M1 A1 B1 A1 **7**	 Equation of BC Must be the one needed for comparison
OR (ii)	Attempt at equation of trajectory $y = 0.577x - 0.221x^2$ $y = -0.577x$ Solving their quadratic and linear equations x = 5.2 (or better) and y = -3.0 (or better) Distance = 6.0 (or better) Ball does not hit the roof	M1 A1 B1 M1 A1 B1 A1 **7**	 Distance from B to point of intersection

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GCE

Mathematics

Advanced GCE **4730**

Mechanics 3

Mark Scheme for June 2010

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Facsimile: 01223 552610
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1	For included angle marked α or for $0.8(10.5 – 8.5\cos\alpha) = 4\cos\beta$ For opposite side marked 4/0.8 (or 4) or for -- $0.8 \times 8.5\sin\alpha = 4\sin\beta$ $8.4^2 + 6.8^2 – 2 \times 8.4 \times 6.8 \cos\alpha = 4^2$ $\alpha = 28.1^o$	M1 A1 A1 M1 A1ft A1 [6]	For triangle with two of its sides marked 0.8 x 10.5 and 0.8 x 8.5 (or 10.5 and 8.5) or for using I = Δmv in one direction. Allow B1 for omission of 0.8 Allow B1 for omission of 0.8 For using the cosine rule or for eliminating β ft 0.8 mis-used or not used
2(i)	[$100a = 2aV_B$] Vertical component at B is 50 N Vertical component at C is 150 N	M1 A1 A1 [3]	For taking moments about A for AB
(ii)	$100(0.5a) + (\sqrt{3}\ a)F = 150a$ or $100a + 100(1.5a) = 150a + (\sqrt{3}\ a)F$ Frictional force is 57.7 N Direction is to the right	M1 A1ft A1 B1 [4]	For taking moments about B for BC (3 terms needed) or about A for the whole (4 terms needed)
3(i)	u = 4 v = 2	B1 B1 [2]	
(ii)	mu = ma + mb (or u = b – a) u = b – a (or mu = ma + mb) a = 0 and b = 4ms^{-1} Speed of A is 2ms^{-1} and direction at 90^o to the wall Speed of B is 4ms^{-1} and direction parallel to the wall	M1 A1 B1 A1ft A1ft A1ft [6]	For using the principle of conservation of momentum or for using NEL with e = 1 ft incorrect u ft incorrect v ft incorrect u
4(i)	[$0.25\ dv/dt = 3/50 – t^2/2400$] $v = 12t/50 – t^3/1800$ [v(12) = 1.92] [$0.25\ dv/dt = t^2/2400 - 3/50$ ➔ $v = t^3/1800 - 12t/50 + C_2$] [$1.92 = 0.96 – 2.88 + C_2$] $v = t^3/1800 - 12t/50 + 3.84$ $v(24) = 5.76 = 3 \times v(12)$	M1 M1 A1 M1 M1 M1 A1 A1 [8]	For using Newton's second law (1st or 2nd stage) For attempting to integrate (1st stage) and using v(0) = 0 (may be implied by the absence of + C_1) For evaluating v when force is zero For using Newton's second law (2nd stage) and integrating For using v(12) = 1.92 AG

(ii)	Sketch has $v(0) = 0$ and slope decreasing (convex upwards) for $0 < t < 12$ Sketch has slope increasing (concave upwards) for $12 < t < 24$ Sketch has $v(t)$ continuous, single valued and increasing (except possibly at $t = 12$) with $v(24)$ seen to be $> 2v(12)$	B1 B1 B1 [3]	
5(i)	For using amplitude as a coefficient of a relevant trigonometric function. For using the value of ω as a coefficient of t in a relevant trigonometric function. $x_1 = 3\cos t$ and $x_2 = 4\cos 1.5t$	B1 B1 B1 [3]	
(ii)	Part distance is 20m [20 – (-3.62)] Distance travelled by P_2 is 23.6 m	M1 A1 M1 A1 [4]	For using distance travelled by P_2 for $0 < t < 5\pi/3$ is $5A_2$ For subtracting displacement of P_2 when t = 5.99 from part distance.
(iii)	$\dot{x}_1 = -3\sin t$; $\dot{x}_2 = -6\sin 1.5t$ $v_1 = 0.867$, $v_2 = -2.55$; opposite directions	M1 A1 M1 A1 [4]	For differentiating x_1 and x_2 For evaluating when t = 5.99 (must use radians)
	Alternative for (iii): $v_1^2 = 3^2 - 2.87^2$, $v_2^2 = 2.25[4^2 - (-3.62)^2]$ [$\pi < 5.99 < 2\pi \Rightarrow v_1 > 0$, $4\pi/3 < 5.99 < 2\pi \Rightarrow v_2 < 0$] $v_1 = 0.867$, $v_2 = -2.55$; opposite directions	M1 A1 M1 A1	For using $v^2 = n^2(a^2 - x^2)$ (must use radians to find values of x) For using the idea that v starts –ve and changes sign at intervals of T/2 s
6(i)	PE loss at lowest allowable point = 25W EE gain = $32000 \times 5^2/(2 \times 20)$ [25W = 20000] Value of W is 800	B1 M1 A1 M1 A1 [5]	For using $EE = \lambda x^2/(2L)$; may be scored in (i) or in (ii) For equating PE loss and EE gain and attempting to solve for W
(ii)	[$800 = 32000x/20$] $\frac{1}{2}(800/9.8)v^2 = 800 \times 20.5 - 32000 \times 0.5^2/(2 \times 20)$ Maximum speed is 19.9ms^{-1}	M1 M1 A1 A1 [4]	For using $W = \lambda x/L$ at max speed For using the principle of conservation of energy (3 terms required)
(iii)	$(800)\ddot{x}/g = 800 - 32000 \times 5/20$ Max. deceleration is 88.2 ms^{-2}	M1 A1 A1 [3]	For applying Newton's second law to jumper at lowest point (3 terms needed)

7(i)	[½ mv^2 – ½ m 6^2 = mg(0.7)] Speed of P before collision is 7.05ms^{-1} Coefficient of restitution is 0.695	M1 A1 B1ft [3]	For using the principle of conservation of energy for P (3 terms needed) ft 4.9 ÷ speed of P before collision
(ii)	[½ mv^2 = ½ m 4.9^2 - mg0.7(1 - cosθ)] v^2 = 3.43(3 + 4 cosθ) T – mgcosθ = mv^2/0.7 [T – m9.8cosθ = m3.43(3 + 4 cosθ)/0.7] Tension is 14.7m(1 + 2cosθ)N	M1 A1 M1 A1 M1 A1 [6]	For using the principle of conservation of energy for Q Accept any correct form For using Newton's second law radially with $a_r = v^2/r$ For substituting for v^2 AG
(iii)	T = 0 ➔ θ = 120° Radial acceleration is (±)4.9 ms^{-1} **or** transverse acceleration is (±)8.49 ms^{-1} Radial acceleration is (±)4.9 ms^{-1} **and** transverse acceleration is (±)8.49 ms^{-1}	B1 M1 A1 B1 [4]	For using a_r = -gcosθ {or 3.43(3 + 4cosθ)/0.7} or a_t = –gsinθ
			SR for candidates with a sin/cos mix in the work for M1 A1 B1 immediately above. (max. 1/3) Radial acceleration is (±)8.49 ms^{-1} **and** transverse acceleration is (±)4.9 ms^{-1} B1
(iv)	[V^2 = 3.43{3 + 4(-0.5)}x0.5^2 or V^2 = (-gcos120° x 0.7) x $\cos^2 60°$] V^2 = 0.8575 [mgH = ½ m(4.9^2 – 0.8575) or mg(H – 1.05) = ½ m(3.43 – 0.8575)] Greatest height is 1.18 m	M1 A1 M1 A1 [4]	For using V = v(120°) x cos60° AG For using the principle of conservation of energy

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GCE

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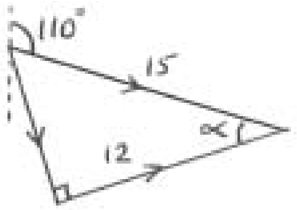
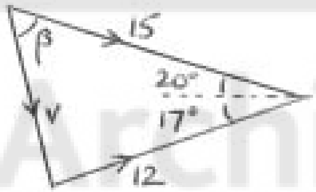
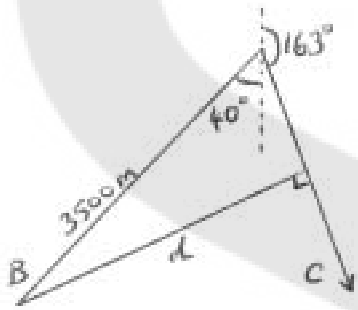
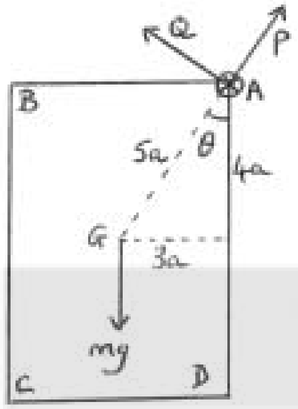
1 **(i)**	Using $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$, $1020 = 80 \times 15 + \frac{1}{2}\alpha \times 15^2$ $\alpha = -1.6$ Angular deceleration is 1.6 rad s^{-2}	M1 A1 **[2]**	
(ii)	Using $\theta = \omega_2 t - \frac{1}{2}\alpha t^2$, $\theta = 0 - \frac{1}{2} \times (-1.6) \times 5^2$ Angle is 20 rad	M1 A1 ft **[2]**	ft is $12.5\|\alpha\|$
(iii)	Using $\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta$, $0 = 80^2 + 2 \times (-1.6)\theta$ $\theta = 2000$ Number of revolutions is 318 (3 sf)	M1 A1 ft A1 **[3]**	*Accept* $\frac{1000}{\pi}$
2	Area is $\int_0^{\ln 3} \mathrm{e}^{-x}\,\mathrm{d}x$ $= \left[-\mathrm{e}^{-x} \right]_0^{\ln 3} \quad (= \frac{2}{3})$ $\int x\,y\,\mathrm{d}x = \int_0^{\ln 3} x\,\mathrm{e}^{-x}\,\mathrm{d}x$ $= \left[-x\,\mathrm{e}^{-x} - \mathrm{e}^{-x} \right]_0^{\ln 3} \quad (= \frac{2}{3} - \frac{1}{3}\ln 3)$ $\bar{x} = \frac{\frac{2}{3} - \frac{1}{3}\ln 3}{\frac{2}{3}} = 1 - \frac{1}{2}\ln 3$ $\int \frac{1}{2}y^2\,\mathrm{d}x = \int_0^{\ln 3} \frac{1}{2}(\mathrm{e}^{-x})^2\,\mathrm{d}x$ $= \left[-\frac{1}{4}\mathrm{e}^{-2x} \right]_0^{\ln 3} \quad (= \frac{2}{9})$ $\bar{y} = \frac{\frac{2}{9}}{\frac{2}{3}} = \frac{1}{3}$	M1 A1 M1 M1 A1 A1 M1 A1 A1 **[9]**	*Limits not required* For $-\mathrm{e}^{-x}$ *Limits not required* Integration by parts For $-x\,\mathrm{e}^{-x} - \mathrm{e}^{-x}$ $\int (\mathrm{e}^{-x})^2\,\mathrm{d}x$ or $\int (-\ln y)y\,\mathrm{d}y + (\frac{1}{3}\ln 3) \times \frac{1}{6}$ $-\frac{1}{4}\mathrm{e}^{-2x}$ or $-\frac{1}{2}y^2 \ln y + \frac{1}{4}y^2$ *(dep on M1)* *Max penalty of 1 mark for correct answers in an unacceptable form (eg decimals)*
3 **(i)**	By conservation of angular momentum $I_2 \times 15 = 0.9 \times 16$ $I_2 = 0.96$ $I_2 = 0.9 + m \times 0.4^2$ Mass is 0.375 kg	M1 A1 M1 A1 **[4]**	Using $I\omega$
(ii)	KE before is $\frac{1}{2} \times 0.9 \times 16^2$ KE after is $\frac{1}{2} \times 0.96 \times 15^2$ Loss of KE is $115.2 - 108 = 7.2 \text{ J}$	M1 A1 ft A1 **[3]**	Using $\frac{1}{2}I\omega^2$ Both expressions correct

4 (i)	 $\cos\alpha = \frac{12}{15}$ $\alpha = 36.87°$ (4 sf) Bearing of $\mathbf{v}_B$ is $110 - 36.87 = 073.13$ $= 073°$ (nearest degree)	M1 A1 M1 A1 ag [4]	Velocity triangle with $90°$ opposite $\mathbf{v}_C$ Correct velocity triangle Finding a relevant angle
(ii)	Magnitude is $\sqrt{15^2 - 12^2} = 9\ \mathrm{m\,s^{-1}}$ Direction is $90°$ from $\mathbf{v}_B$ Bearing is $73.13 + 90 = 163°$ (nearest degree)	B1 M1 A1 [3]	Accept 8.95 to 9.05
	Alternative for **(ii)** (*using given answer in* **(i)**)  $v^2 = 12^2 + 15^2 - 2\times 12\times 15\cos 37°$ $v = 9$ $\frac{\sin\beta}{12} = \frac{\sin 37°}{v}$ $\beta = 53°$ Bearing is $110 + 53 = 163°$	B1 M1 A1	or Relative velocity is $\begin{pmatrix} v\sin\theta \\ v\cos\theta \end{pmatrix} = \begin{pmatrix} 15\sin 110 \\ 15\cos 110 \end{pmatrix} - \begin{pmatrix} 12\sin 73 \\ 12\cos 73 \end{pmatrix} \approx \begin{pmatrix} 2.6 \\ -8.6 \end{pmatrix}$ or $v^2 = (2.6...)^2 + (-8.6...)^2$ Accept 8.95 to 9.05 Finding a relevant angle or $\tan\theta = \frac{2.6...}{-8.6...}$
(iii)	As viewed from *B*  $d = 3500\sin 56.87°$ Shortest distance is 2930 m (3 sf)	M1 M1 A1 [3]	Diagram indicating initial displacement and relative velocity *May be implied* Accept 2910 to 2950
	Alternative for **(iii)** $d^2 = (3500\sin 40° + 2.6...t)^2$ $+ (3500\cos 40° - 8.6...t)^2$ Minimum when $-34432 + 162t = 0$ $t = 213$ Shortest distance is 2930 m (3 sf)	M1 M1 A1	Differentiating or completing the square Accept 2910 to 2950

5 (i)	$I = \int_{-a}^{5a} \frac{m}{6a} x^2 \, dx$ or $\int_{-a}^{5a} \rho x^2 \, dx$	M1 M1 A1	$(\delta m)x^2$ or $(\rho \delta x)x^2$ or integrating x^2 Using $\delta m = \frac{m \delta x}{6a}$ or $\rho = \frac{m}{6a}$ Correct integral expression for *I* eg $I = \int_0^{5a} ... + \int_0^{a} ...$ $I = \int_{-3a}^{3a} ... + m(2a)^2$, $I = 2\int_0^{3a} ... + m(2a)^2$ $I = \int_0^{6a} ... - m(3a)^2 + m(2a)^2$
	$= \left[\frac{m}{18a} x^3 \right]_{-a}^{5a} = \frac{m}{18a}(125a^3 + a^3)$ or $42\rho a^3$ $= \frac{126ma^3}{18a} = 7ma^2$	M1 A1 ag **[5]**	Evaluating definite integral *Dependent on integrating* x^2
(ii)	WD by couple is $\frac{6mga}{\pi} \times 3\pi$ $(= 18mga)$ Gain of PE is $mg(4a)$ $18mga = 4mga + \frac{1}{2}(7ma^2)\omega^2$ Angular speed is $\sqrt{\frac{4g}{a}}$	M1 A1 B1 M1 A1 ft A1 **[6]**	Using $C\theta$ Equation involving WD, PE and $\frac{1}{2} I \omega^2$

6 (i)	$\frac{dV}{d\theta} = mga(3\cos\theta + 4\sin\theta - 3)$ When $\theta = 0$, $\frac{dV}{d\theta} = mga(3+0-3) = 0$ so $\theta = 0$ is a position of equilibrium $\frac{d^2V}{d\theta^2} = mga(-3\sin\theta + 4\cos\theta)$ When $\theta = 0$, $\frac{d^2V}{d\theta^2} = 4mga > 0$ hence the equilibrium is stable	B1 M1 A1 ag M1 A1 ag **[5]**	Considering $\frac{dV}{d\theta} = 0$ Correctly shown Considering $\frac{d^2V}{d\theta^2}$ (or other method) $V'' = 4mga \Rightarrow \text{Stable}$ M1A0 $V'' = 4mga \Rightarrow \text{Minimum} \Rightarrow \text{Stable}$ M1A1
(ii)	Speed of *P* and *Q* is $a\dot\theta$ KE is $\frac{1}{2}(5m)(a\dot\theta)^2 + \frac{1}{2}(3m)(a\dot\theta)^2$ or $\frac{1}{2}(8m)(a\dot\theta)^2$ $= \frac{5}{2}ma^2\dot\theta^2 + \frac{3}{2}ma^2\dot\theta^2$ $= 4ma^2\dot\theta^2$	M1 A1 ag **[2]**	Or moment of inertia of *P* is $5ma^2$ $\frac{5}{2}ma^2\dot\theta^2 + \frac{3}{2}ma^2\dot\theta^2$ M1A1 $\frac{1}{2}(5ma^2)\dot\theta^2 + \frac{1}{2}(3ma^2)\dot\theta^2$ M1A0 $\frac{1}{2}(8ma^2)\dot\theta^2$ M1A0
(iii)	$V + 4ma^2\dot\theta^2 = K$ $\frac{dV}{d\theta}\dot\theta + 8ma^2\dot\theta\ddot\theta = 0$ $mga(3\cos\theta + 4\sin\theta - 3)\dot\theta + 8ma^2\dot\theta\ddot\theta = 0$ For small θ, $\sin\theta \approx \theta$, $\cos\theta \approx 1$ $mga(3 + 4\theta - 3) + 8ma^2\ddot\theta \approx 0$ $\ddot\theta \approx -\frac{g}{2a}\theta$ Approximate period is $2\pi\sqrt{\frac{2a}{g}}$	M1 A1 M1 A1 ft A1 **[5]**	$= 0$ is required for A1 *(may be implied by later work)* Linear approximation (ft is dep on M1M1)

7 (i)	$I = \frac{1}{3}m\{(3a)^2 + (4a)^2\} + m(5a)^2$ $= \dfrac{100ma^2}{3}$	M1 A1 A1 [3]	Using parallel (or perpendicular) axes rule or $I = \frac{4}{3}m(3a)^2 + \frac{4}{3}m(4a)^2$
(ii)	 By conservation of energy, $\frac{1}{2}(\frac{100}{3}ma^2)\omega^2 = mg(4a - 3a)$ $\frac{50}{3}ma^2\omega^2 = mga$ Angular speed is $\sqrt{\dfrac{3g}{50a}}$ $-mg(3a) = (\frac{100}{3}ma^2)\alpha$ Angular acceleration is $(-)\dfrac{9g}{100a}$	M1 A1 ft A1 ag M1 A1 [5]	Equation involving KE and PE Using $C = I\alpha$
(iii)	$P - mg\cos\theta = m(5a)\omega^2$ $P - \frac{4}{5}mg = m(5a)\left(\dfrac{3g}{50a}\right)$ $P = \frac{11}{10}mg$ $Q - mg\sin\theta = m(5a)\alpha$ $Q - \frac{3}{5}mg = -m(5a)\left(\dfrac{9g}{100a}\right)$ $Q = \frac{3}{20}mg$ $F = \sqrt{P^2 + Q^2} = \frac{1}{20}mg\sqrt{22^2 + 3^2}$ $= \dfrac{\sqrt{493}}{20}mg$	M1 A2 M1 A2 ft M1 A1 ag [8]	Equation involving *P* and $r\omega^2$ Give A1 if correct apart from sign(s) *(Allow $\frac{3}{5}H + \frac{4}{5}V$ in place of P)* Equation involving *Q* and $r\alpha$ Give A1 if correct apart from sign(s) ft for wrong value of α ft for wrong value of *r* in second equation *(Allow $\frac{3}{5}V - \frac{4}{5}H$ in place of Q)* *Dependent on previous M1M1*
	Alternative for **(iii)** $H = m(5a)\omega^2\sin\theta - m(5a)\alpha\cos\theta$ $H = m(5a)\left(\frac{3g}{50a}\right)\left(\frac{3}{5}\right) + m(5a)\left(\frac{9g}{100a}\right)\left(\frac{4}{5}\right)$ $V - mg = m(5a)\omega^2\cos\theta + m(5a)\alpha\sin\theta$ $V - mg = m(5a)\left(\frac{3g}{50a}\right)\left(\frac{4}{5}\right) - m(5a)\left(\frac{9g}{100a}\right)\left(\frac{3}{5}\right)$ $H = \frac{27}{50}mg,\quad V = \frac{97}{100}mg$	M1 A2 ft M1 A2 ft	Equation involving *H*, $r\omega^2$ and $r\alpha$ Give A1 if correct apart from sign(s) Equation involving *V*, $r\omega^2$ and $r\alpha$ Give A1 if correct apart from sign(s)

	$F=\sqrt{H^2+V^2}=\frac{1}{100}mg\sqrt{54^2+97^2}$ $=\frac{\sqrt{12325}}{100}mg=\frac{\sqrt{493}}{20}mg$	M1 A1 ag	*Dependent on previous M1M1*

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1 Hills Road
Cambridge
CB1 2EU

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RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **4732**

Probability and Statistics 1

Mark Scheme for June 2010

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Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to $\geq$ 3sfs, ISW for later rounding
Penalise over-rounding only once in paper.

1i	590	B1 1	Allow approximately 590
ii	Graph horiz (for $\geq$ 55 mks) oe	B1 1	or levels off, or grad = 0, grad not increase Allow line not rise, goes flat, plateaus, stops increasing, not increase, doesn't move
iii	39 to 41	B1 1	
iv	Attempt read cf at 26 or 27 Double & attempt read x Max C = 29 to 31.5	M1 M1 A1 3	eg 26 mks → 150th 27 mks → 180th eg read at cf = 300 or 360 Indep of first M1 May be implied by ans Answer within range, no working, M1M1A1 32 without working, sc B1
v	LQ = 25.5-26.5 or UQ = 34-35.5 IQR = 8-10 (German) more spread	M1 A1 B1ft 3	M1 for one correct quartile dep $\geq$ 1 correct quartile or no working or less consistent, less uniform, less similar, more varied, more variable, greater variance, more spaced apart, further apart ft their IQR; must be consistent with IQR Correct comment with no working: M0A0B1
Total		9	
2i	Opposite orders or ranks or scores or results or marks $r_s = -1$	B1 1	or reversed, or backwards, or inverse or as one increases the other decreases Needs reason AND value
ii	Attempt Σd^2 (= 6) $1 - \frac{6\times\Sigma d^2}{3(3^2-1)}$ $= -\frac{1}{2}$ oe	M1 M1 A1 3	 dep 1st M1 Allow use wrong table for M1M1
iii	3! or ${}^3P_3$ or 6 1 ÷ their '6' $\frac{1}{6}$ oe eg $\frac{6}{36}$	M1 M1 A1 3	r attempt list possible orders of 1,2,3 ($\geq$3 orders) 2nd M1 for fully correct method only or $\frac{1}{3}\times\frac{1}{2}(\times 1)$: M1M1
Total		7	
3i	If x is contr (or indep) or y depend't, use y on x If neither variable contr'd (or indep) AND want est y from x: use y on x	B1 B1 2	Allow x increases constantly, is predetermined, you choose x, you set x, x is fixed, x is chosen Allow y not controlled AND want est y from x Ignore incorrect comments
iia	$S_{xx} = 510000 - \frac{1800^2}{9}$ (= 150000) $S_{xy} = 4080 - \frac{1800\times14.4}{9}$ (= 1200) $b = \frac{'1200'}{'150000'}$ (= 0.008) $y - \frac{14.4}{9} = 0.008(x - \frac{1800}{9})$ $y = 0.008x$ (+ 0)	 M1 M1 M1 A1 4	or $\frac{510000}{9} - 200^2$ (= 16666.7) or $\frac{4080}{9}$ - 200×1.6 (= 133.33) M1 for either S $b = \frac{'133.33'}{'16666.7'}$ dep correct expressions both S's or $a = \frac{14.4}{9} - 0.008\times\frac{1800}{9}$ (= 0) Must be all correct for M1 CAO
iib	312.5 or 313	B1ft 1	ft their equn in (iia)
iic	–0.4	B1ft 1	ft their equn in (iia)

iid	Contraction oe Unreliable because extrapolated oe	B1(ft) B1 2	or length decreased, shorter, pushed in, shrunk, smaller or not in the range of x or not in range of previous results
Total		**10**	
4ia	0.299 (3 sf)	B1 1	
ib	0.2991 – 0.1040 = 0.195 (3 sf) or $\frac{1280}{6561}$ oe	M1 A1 2	Must subtract correct pair from table
iia	$^{15}C_4\times(1\text{-}0.22)^{11}\times0.22^4$ = 0.208 (3 sf)	M1 A1 2	Allow M1 for $^{15}C_4\times0.88^{11}\times0.22^4$
iib	$(15 \times 0.22 =)$ 3.3 $15\times0.22\times(1–0.22)$ or ‘3.3’$\times(1–0.22)$ = 2.57 (3 sf)	B1 M1 A1 3	 Allow M1 for $15 \times 0.22 \times 0.88$
Total		**8**	
5i	$\frac{1}{2}\times\frac{1}{3}$ or $\frac{2}{4}\times\frac{1}{3}$ or $\frac{1}{^4C_2}$ or $\frac{2}{12}$ (= $\frac{1}{6}$ **AG**) $\frac{1}{4}\times\frac{2}{3}$ or $2\times\frac{1}{4}\times\frac{1}{3}$ or $\frac{1}{2}\times\frac{1}{3}$ or $\frac{2}{4}\times\frac{1}{3}$ Add two of these or double one (= $\frac{1}{3}$ **AG**)	B1 B1 B1 3	or 1 out of 6 or 2 out of 12 or $\frac{2!}{4!}\times2$ or $\frac{2}{12}$ or $\frac{1}{6}$ or $\frac{1}{3!}$ or $\frac{1}{^4C_2}$ or $\frac{2!}{4!}\times2$ or $\frac{2}{^4C_2}$ or $4\times\frac{1}{4}\times\frac{1}{3}$ or $\frac{2}{4}\times\frac{2}{3}$ or $\frac{4}{12}$ or $\frac{2!}{4!}\times4$ B1B1 or $\frac{2}{6}$ or $2\times\frac{1}{6}$ or $\frac{2}{3!}$ or $\frac{2!}{3!}$ B1B1
ii	X = 3, 4, 5, 6 only, stated or used P(X =5) wking as for P(X =4) above or $1 – (“\frac{1}{6}” + \frac{1}{3} + \frac{1}{6})$ or $\frac{1}{3}$ P(X =3) wking as for P(X =6) above or $1 – (\frac{1}{3} + “\frac{1}{3}” + \frac{1}{6})$ or $\frac{1}{6}$ 3 4 5 6 $\frac{1}{6}$ $\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{6}$ oe	B1 M1 M1 A1 4	Allow repetitions Allow other values with zero probabilities. or M1 for total of their probs = 1, dep B1 or P(X =3)= $\frac{1}{6}$, P(X = 4)= $\frac{1}{3}$, P(X = 5)= $\frac{1}{3}$, P(X = 6)= $\frac{1}{6}$ Complete list of values linked to probs
iii	Σxp = $4\frac{1}{2}$ Σx^2p (= $21\frac{1}{6}$) – ‘$4\frac{1}{2}$’^{2} = $\frac{11}{12}$ or 0.917 (3 sf)	M1 A1 M1 M1 A1 5	$\geq$ 2 terms correct ft $\geq$ 2 terms correct ft Independent except dependent on +ve result
Total		**12**	

6	$m = (9 \times 6 + 3) \div 10$ $= 5.7$ $2 = \frac{\Sigma x^2}{9} - 6^2$ $\Sigma x^2 = 2\times9 + 6^2\times9$ or 342 $v = \frac{('342' + 3^2)}{10} - '5.7'^2$ $= 2.61$ oe	M1 A1 M1 A1 M1 A1 6	or ((Sum of any 9 nos totalling 54) + 3) ÷ 10 or $\frac{\Sigma(x-6)^2}{9} = 2$ M1 or $\Sigma x^2 = 18 + 12\times54 - 36\times9$ or 342 A1 dep Σx^2 attempted, eg $(\Sigma x)^2$ (= 3249) or just state 'Σx^2'; allow √ CAO
Total		**6**	
7i	${}^4C_2 \times {}^6C_3 \times {}^5C_4$ or $6\times20\times5$ $= 600$	M1M1 A1 3	M1 for any 2 correct combs seen, even if added
ii	$\frac{2}{4}$ or $\frac{{}^3C_1}{{}^4C_2}$ or $\frac{{}^3C_1\times{}^6C_3\times{}^5C_4}{{}^4C_2\times{}^6C_3\times{}^5C_4}$ or $\frac{{}^3C_1\times{}^6C_3\times{}^5C_4}{'600'}$ $= \frac{1}{2}$ oe	M1 A1 2	or $\frac{1}{4}\times1+\frac{3}{4}\times\frac{1}{3}$ or $\frac{1}{4}\times 2$ or $\frac{1}{4}+\frac{1}{4}$
iii	${}^3C_1 \times {}^6C_3\ (\times\ {}^4C_4) + {}^3C_2 \times {}^6C_3 \times {}^5C_4$ 360	M1M1 A1 3	M1 either product seen, even if × or ÷ by something
Total		**8**	

8			
8ia	Geo(0.3) stated or implied $0.7^3 \times 0.3$ = 0.103 (3 sf)	M1 M1 A1 3	by $0.7^n \times 0.3$
b	0.7^3 or 0.343 $1 - 0.7^3$ = 0.657	M1 M1 A1 3	0.7^3 must be alone, ie not $0.7^3 \times 0.3$ or similar allow $1 - 0.7^4$ or 0.7599 or 0.76 for M1 only or $0.3 + 0.7\times0.3 + 0.7^2\times0.3$: M1M1 1 term wrong or omitted or extra M1 or $1 - (0.3 + 0.7\times0.3 + 0.7^2\times0.3)$ or 0.343: M1
iia	State or imply one viewer in 1st four ${}^4C_1 \times 0.7^3 \times 0.3$ (= 0.412) $\times$ 0.3 = 0.123 (3 sf)	M1 M1 M1 A1 4	or B(4, 0.3) stated, or ${}^4C_1$ used, or YNNNY dep 1st M1
b	$0.7^5 + {}^5C_1 \times 0.7^4 \times 0.3$ = 0.528 (3 sf)	M1 A1 2	or $1-(0.3^2 + 2\times0.3^2\times0.7 + 3\times0.3^2\times0.7^2 + 4\times0.3^2\times0.7)$ Not ISW, eg 1 – 0.528: M1A0
Total		**12**	

Total 72 marks

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GCE

Mathematics

Advanced GCE **4733/01**

Probability and Statistics 2

Mark Scheme for June 2010

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1	(i)(a)	$1 - P(\le 6) = 1 - 0.8675$ $= \mathbf{0.1325}$	M1 A1 **2**	1 – .9361 or 1 – .8786 or 1 – .8558: M1. .9721: M0 Or 0.132 or 0.133
	(b)	Po(0.42) $e^{-0.42}\dfrac{0.42^2}{2!}$ $= \mathbf{0.05795}$	M1 M1 A1 **3**	Po(0.42) stated or implied Correct formula, any numerical λ Answer, art 0.058. Interpolation in tables: M1B2
	(ii)	E.g. "Contagious so incidences do not occur independently", or "more cases in winter so not at constant average rate"	B2 **2**	Contextualised reason, referred to conditions: B2. No marks for mere learnt phrases or spurious reasons, e.g. not just "independently, singly and constant average rate". See notes.
2	(i)	B(10, 0.35) $P(< 3)$ $= \mathbf{0.2616}$	M1 M1 A1 **3**	B(10, 0.35) stated or implied Tables used, e.g. 0.5138 or 0.3373, or formula ± 1 term Answer 0.2616 or better or 0.262 only
	(ii)	Binomial requires being chosen independently, which this is not, but unimportant as population is large	B2 **2**	Focus on "Without replacement" negating independence condition. It doesn't negate "constant probability" condition but can allow B1 if "selected". See notes
3	(i)	$\left(\dfrac{32-40}{\sigma}\right) = \Phi^{-1}(0.2) = -0.842$ $\sigma = 9.5[06]$	M1 B1 A1 **3**	Standardise and equate to Φ^{-1}, allow "1 –" errors, σ^2, cc 0.842 seen Answer, 9.5 or in range [9.50, 9.51], c.w.o.
	(ii)	B(90, 0.2) $\approx N(18, 14.4)$ $1-\Phi\left(\dfrac{19.5-18}{\sqrt{14.4}}\right) = 1 - \Phi(0.3953)$ $= 1 - 0.6537$ $= \mathbf{0.3463}$	B1 M1 A1 M1 A1 A1 **6**	B(90, 0.2) stated or implied N, their *np* … … variance their *npq*, allow √ errors Standardise with *np* and *npq*, allow √, cc errors, e.g. .396, .448, .458, .486, .472; √*npq* and cc correct Answer, a.r.t. 0.346 [NB: 0.3491 from Po: 1/6]
4		$H_0 : p = 0.4$, $H_1 : p > 0.4$ $R \sim B(16, 0.4)$:	B1 B1 M1	Fully correct, B2. Allow π. *p* omitted or μ used in both, or > wrong: B1 only. *x* or $\bar{x}$ or 6.4 etc: B0 B(16, 0.4) stated or implied, allow N(6.4, 3.84)
	(α)	$P(R \ge 11) = 0.0191$ > 0.01	A1 A1	Allow for $P(\le 10) = 0.9808$, and < 0.99, or $z = 2.092$ or $p = 0.018$, but *not* $P(\le 11) = 0.9951$ or $P(= 11) = 0.0143$ Explicit comp with .01, or $z<2.326$, *not* from ≤11 or =11
	(β)	CR $R \ge 12$ and $11 < 12$ Probability 0.0049	A1 A1	Must be clear that it's ≥ 12 and not ≤ 11 Needs to be seen, allow 0.9951 here, or $p = .0047$ from N
		Do not reject H_0. Insufficient evidence that proportion of commuters who travel by train has increased	M1 A1 FT **7**	Needs like-with-like, $P(R \ge 11)$ or CR $R \ge 12$ Conclusion correct on their *p* or CR, contextualised, not too assertive, e.g. "evidence that" needed. Normal, $z = 2.34$, "reject" [no cc] can get 6/7
5	(i)	(a) $30 + 1.645 \times \dfrac{5}{\sqrt{10}}$ $= 32.6$ Therefore critical region is $\bar{t} > 32.6$	M1 B1 A1 A1 FT **4**	$30 + 5z/\sqrt{10}$, allow ± but not just –, allow √ errors $z = 1.645$ seen, allow – Critical value, art 32.6 " > *c*" or "≥ *c*", FT on *c* provided > 30, can't be recovered. Withhold if not clear which is CR
		(b) $P(\bar{t} < 32.6 \mid \mu = 35)$ $\dfrac{32.6-35}{5/\sqrt{10}}$ $[= -1.5178]$ **0.0645**	M1* dep*M1 A1 **3**	Need their *c*, final answer < 0.5 and $\mu = 35$ at least, but allow answer > 0.5 if consistent with their (i) Standardise their CV with 35 and √10 or 10 Answer in range [0.064, 0.065], or 0.115 from 1.96 in (a)
	(ii)	$(32.6 - \mu) = 0$ $\mu = 32.6$ $20 + 0.6m = 32.6$ $m = \mathbf{21}$	M1 A1 FT M1 A1 **4**	Standardise *c* with μ, equate to Φ^{-1}, can be implied by: μ = their *c* Equate and solve for *m*, allow from 30 or 35 Answer, a.r.t. 21, c.a.o. MR: 0.05: M1 A0 M1, 16.7 A1 FT Ignore variance throughout (ii)

6	(a)	N(24, 24) $1-\Phi\left(\frac{30.5-24}{\sqrt{24}}\right) = 1-\Phi(1.327)$ $= \mathbf{0.0923}$	B1 B1 M1 A1 A1	 **5**	Normal, mean 24 stated or implied Variance or SD equal to mean Standardise 30 with λ and √λ, allow cc or √ errors, e.g. .131 or .1103 ; 30.5 and √λ correct Answer in range [0.092, 0.0925]
	(b)(i)	*p* or *np* [= 196] is too large	B1	**1**	Correct reason, no wrong reason, don't worry about 5 or 15
	(ii)	Consider (200 – *E*) (200 – *E*) ~ Po(4) P(≥ 6) [= 1 – 0.7851] = **0.2149**	M1 M1 M1 A1	 **4**	Consider complement Po(200×0.02) Poisson tables used, correct tail, e.g. 0.3712 or 0.1107 Answer a.r.t. 0.215 only
7		$H_0 : \mu = 56.8$ $H_1 : \mu \neq 56.8$ $\bar{x} = 17085/300 = 56.95$ $\frac{300}{299}\left(\frac{973847}{300}-56.95^2\right)$ = 2.8637…	B2 B1 M1 M1 A1		Both correct One error: B1, but *not* $\bar{x}$, etc 56.95 or 57.0 seen or implied Biased [2.8541] : M1M0A0 Unbiased estimate method, allow if ÷ 299 seen anywhere Estimate, a.r.t. 2.86 [not 2.85]
	(α)	$z = \frac{56.95-56.8}{\sqrt{2.8637/300}} = 1.535$ 1.535 < 1.645 or 0.0624 > 0.05	M1 A1 A1		Standardise with √300, allow √ errors, cc $z \in [1.53, 1.54]$ or $p \in [0.062, 0.063]$, *not* – 1.535 Compare explicitly *z* with 1.645 or *p* with 0.05, or 2*p* > 0.1, not from μ = 56.95
	(β)	CV $56.8 \pm 1.645 \times \sqrt{\frac{2.8637}{300}}$ 56.96 > 56.95	M1 A1 A1 FT		56.8 + *z*σ/√300, needn't have ±, allow √ errors *z* = 1.645 *c* = 56.96, FT on *z*, and compare 56.95 [c_L = 56.64]
		Do not reject H_0; insufficient evidence that mean thickness is wrong	M1 A1 FT	 **11**	Consistent first conclusion, needs 300, correct method and comparison Conclusion stated in context, not too assertive, e.g. "evidence that" needed
8	(i)	$\int_1^\infty kx^{-a}dx = \left[k\frac{x^{-a+1}}{-a+1}\right]_1^\infty$ Correctly obtain *k* = *a* – 1 **AG**	M1 B1 A1	 **3**	Integrate f(*x*), limits 1 and ∞ (at some stage) Correct indefinite integral Correctly obtain given answer, don't need to see treatment of ∞ but mustn't be wrong. Not k^{-a+1}
	(ii)	$\int_1^\infty 3x^{-3}dx = \left[3\frac{x^{-2}}{-2}\right]_1^\infty = 1\tfrac{1}{2}$ $\int_1^\infty 3x^{-2}dx = \left[3\frac{x^{-1}}{-1}\right]_1^\infty - (1\tfrac{1}{2})^2$ Answer ¾	M1 M1 A1 M1 A1	 **5**	Integrate *x*f(*x*), limits 1 and ∞ (at some stage) [x^4 is *not* MR] Integrate x^2f(*x*), correct limits Either μ = 1½ or E(X^2) = 3 stated or implied, allow *k*, *k*/2 Subtract their numerical μ^2, allow letter if subs later Final answer ¾ or 0.75 only, cwo, e.g. not from μ =–1½. [SR: Limits 0, 1: can get (i) B1, (ii) M1M1M1]
	(iii)	$\int_1^2 (a-1)x^{-a}dx = \left[-x^{-a+1}\right]_1^2 = 0.9$ $1-\frac{1}{2^{a-1}} = 0.9,\ 2^{a-1} = 10$ *a* = 4.322	M1* dep*M1 M1 indept A1	 **4**	Equate ∫f(*x*)d*x*, one limit 2, to 0.9 or 0.1. [Normal: 0 ex 4] Solve equation of this form to get 2^{a-1} = number Use logs or equivalent to solve 2^{a-1} = number Answer, a.r.t. 4.32. T&I: (M1M1) B2 or B0

Specimen Verbal Answers

1	α	"Cases of infection must occur randomly, independently, singly and at constant average rate"	B0
	β	Above + "but it is contagious"	B1
	γ	Above + "but not independent as it is contagious"	B2
	δ	"Not independent as it is contagious"	B2
	ε	"Not constant average rate", or "not independent"	B0
	λ	"Not constant average rate because contagious" *[needs more]*	B1
	ζ	"Not constant average rate because more likely at certain times of year"	B2
	μ	Probabilities changes because of different susceptibilities	B0
	ν	Not constant average rate because of different susceptibilities	B2
	η	Correct but with unjustified or wrong extra assertion *[scattergun]*	B1
	θ	More than one correct assertion, all justified	B2
	π	Valid reason (e.g. "contagious") but not referred to conditions	B1

[Focus is on explaining why the required assumptions might not apply. No credit for regurgitating learnt phrases, such as "events must occur randomly, independently, singly and at constant average rate, even if contextualised.]

2	Don't need either "yes" or "no".		
	α	"No it doesn't invalidate the calculation" *[no reason]*	B0
	β	"Binomial requires not chosen twice" *[false]*	B0
	γ	"Probability has to be constant but here the probabilities change"	B0
	δ	Same but "probability <u>of being chosen</u>" *[false, but allow B1]*	B1
	ε	"Needs to be independently chosen but probabilities change" *[confusion]*	B0
	ζ	"Needs to be independent but one choice affects another" *[correct]*	B2
	η	"The sample is large so it makes little difference" *[false]*	B0
	θ	"The population is large so it makes little difference" *[true]*	B2
	λ	Both correct and wrong reasons (scattergun approach)	B1

[Focus is on modelling conditions for binomial: On every choice of a member of the sample, each member of the population is equally likely to be chosen; and each choice is independent of all other choices.
Recall that in fact even without replacement the probability that any one person is chosen is the same for each choice. Also, the binomial "independence" condition <u>does</u> require the possibility of the same person being chosen twice.]

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GCE

Mathematics

Advanced GCE **4734**

Mark Scheme for June 2010

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1(i)	Total has Poisson distribution with mean λ = 0.21×5+0.24×5 = 2.25 P(≥2) = 1 – $e^{-\lambda}$(1+λ) =0.657	M1 A1 M1 A1 **4**	With ×5 λ or 1+λ in brackets (their λ) Or interpolation from tables
(ii)	EITHER: Each length is a random sample OR: Flaws occur independently on the reels	B1 **1** **[5]**	In context Accept randomly
2	H_0: μ =(or ≥) 170 , H_1: μ < 170 $\overline{x}$ =167.5 s^2 = 5.9 EITHER: (α) (167.5 – 170)/√(5.9/6) = - 2.52(1) Compare with – 2.015 OR: (β) 170 – t√(5.9/6) = 168.0 Compare 167.5 with CV and reject H_0 There is sufficient evidence at the 5% significance level that the machine dispenses less than 170 ml on average.	B1 B1 B1 M1 A1 M1 M1 A1 M1 A1 **[7]**	For both hypotheses; accept words SR 2-tail test: B0B1B1M1A1M1A0 Max 5/7 Standardise 167.5; + or – for M; /6 seen Explicitly Allow 2.571 Finding critical value or region. With t= 2.015 or 2.571 Explicitly. Allow correct use of | t| M0 if z used SR: B1 if no explicit comparison but conclusion "correct"
3(i)	H_0: There is no association between the area in which a shopper lives and the day they shop (H_1: All alternatives) E-Values 27.3 14. 7 37.7 20.3 χ^2= $(4.3-0.5)^2(27.3^{-1}+37.7^{-1}+14.7^{-1}+20.3^{-1})$ = 2.606 Compare with 2.706 Do not reject H_0. There is insufficient evidence of an association. SR: If H_0 association, lose 1st B1 and last M1A1	B1 M1 A1 M1 ft A1 A1 M1 A1 **8**	SR difference in proportions B1 define and evaluate p_1 and p_2 with H_0 B1 for p=0.42 M1A1 for z = ±1.827 or 1.835(no pe) M1A0 Max 5/8 At least one E value correct (M1) All correct(A1) At least one χ^2, no or wrong cc, (M1FtE) All correct (A1); 2.606 or 2.61 (A1) Or use calculator (p = 0.106) SR: B1 if no explicit comparison, as Q2 SR: If H_0 association, lose 1st B1 and last M1A1
(ii)	Conclusion the same since critical value > 2.706 (and test statistic unchanged)	B1 **1** **[9]**	OR from z=±2.17, SR

4(i)	s^2= (1183.65-246.6^2/70)/69 Use $\bar{x} \pm zs/\sqrt{(70)}$ $s/\sqrt{(70)}$ 1.645 (3.10, 3.94)	M1 M1 A1 A1 A1 **5**	AEF Allow without ft or with s^2; with 70 Their s A0 if interval not indicated
(ii)	Change 90 to around 90	B1 **1**	Or equivalent
(iii)	$4(0.9)^3(0.1) + 0.9^4$ =0.9477	M1 A1 **2** **[8]**	Use of bino with p=0.9 or 0.1 and 4 and Correct terms considered. art 0.948
5(i)	$e^{-2.25} - e^{-4}$ × 150 = 13.1 Last: 150 – sum=2.7	M1 A1 A1 A1 ft **4**	Or find last entry using F(x) Or 2.7 if found first Or 13.1 any accuracy
(ii)	(H_0: Data fits the model, H_1: Data does not fit) Combine last two cells $\chi^2 = 7.8^2/33.2 + 11.6^2/61.6 + 7.4^2/39.4 + 11.2^2/15.8$ = 13.3(46) Compare with 9.348 (or 11.14) , reject H_0 **(**There is sufficient evidence at the $2\frac{1}{2}$ % significance level that) the model is not a good fit	B1 M1***Dep** A1 A1 M1 A1 ft Dep* **6** **[10]**	At least two correct All correct In range 13.2 to 13.5 SR: If last 2 cells are not combined B0M1A1A1(for 13. 5) M1A1 If no explicit comparison B1 if conclusion follows
6(i)	Anxiety scores; have normal distributions; common variance; independent samples H_0: $\mu_E = \mu_C$, H_1: $\mu_E < \mu_C$ s^2 = (1923.56+1147.58)/29 (= 105.9) (t) = $(32.16 - 38.21)/\sqrt{[105.9(18^{-1}+13^{-1})]}$ = - 1.615 t_{crit} = - 1.699 Compare -1.615 with -1.699 and do not reject H_0 There is insufficient evidence at the 5% significance level to show that anxiety is reduced by listening to relaxation tapes	B2 B1 B1 M1 A1 A1 B1 M1 A1 ft **10**	Context + 2 valid points B2 Context + 1VP, no context +2VP B1 Not in words Allow 1 error; eg s^2= 1923.56/(17or18) + All correct 47.5/(12or13) Or **+** Or **+;** accept art ±1.70 Or **+ , +.** M0 if t not ±1.699,±2.045 In context, not over-assertive OR Find CV or CR: B2B1B1; C= or ≥ st , t = ±1.699 or ±2.015 M1A1 t= ±1.699 B1; G= 6.11(2) A1; 6.112> 6.05 and reject H_0 etcM1A1
(ii)	Sample sizes are too small (to appeal to CLT)	B1 **1** **[11]**	

7(i)	Use $\sum F + \sum M \sim N(\mu, \sigma^2)$ $\mu = 1104.9$ $\sigma^2 = 6\times9.3^2 + 9\times8.5^2$ $= 1169.2$ P(> 1150) = 1 – Φ([1150 – 1104.9]/$\sqrt{}$(1169.2) = 0.0937	M1 A1 M1 A1 M1 A1 **6**	Sum of indep normal variables is normal Standardise, correct tail. M0 $\sigma/\sqrt{15}$ Accept .094
(ii)	If unknown M, prob $\frac{1}{2}$, 6F and 9M as before. If unknown W, prob $\frac{1}{2}$, 7W and 8M Having N(1093.3,1183.4) P(> 1150)= 1 – Φ(1.648) = 0.0497 P = $\frac{1}{2}\times 0.0936 + \frac{1}{2}\times 0.0497$ = 0.07165	M1 B1 B1 A1 M1 A1 **6** **[12]**	Considering two cases Mean and variance Use of $\frac{1}{2}$ ART 0.072
8(i)	$X = \frac{1}{4}S^2$ $F(s) = \int_1^s \frac{8}{3s^3}\,ds = \left[-\frac{4}{3s^2}\right]_1^s$ $= \frac{4}{3}(1 - 1/s^2)$ G(x) = P($X \le x$) = P($S \le 2\sqrt{x}$) = F(2$\sqrt{x}$) $= \frac{4}{3} - \frac{1}{3x}$ $g(x) = \begin{cases} \frac{1}{3x^2} & \frac{1}{4} \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$	B1 M1 A1 M1 A1 ft M1 B1 **7**	 Ignore range here SR: B1 for G(x)=F(2$\sqrt{x}$) without justification and with correct result ft F For G′ (a) For range
(ii)	EITHER: G(m) = $\frac{1}{2}$ $\Rightarrow \frac{4}{3} - \frac{1}{3x} = \frac{1}{2}$ $\Rightarrow m = \frac{2}{5}$ OR: $\int_{1/4}^{m} \frac{1}{3x^2}\,dx = \frac{1}{2}$ $\Rightarrow \left[-\frac{1}{3x}\right]_{1/4}^{m} = \frac{1}{2}$ $\Rightarrow \quad m = \frac{2}{5}$	M1 A1 ft A1 M1 A1 A1 **3** **[10]**	ft G(x) in **(i)** CAO Allow wrong $\frac{1}{4}$ Allow wrong $\frac{1}{4}$ CAO

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RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **4735**

Mark Scheme for June 2010

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1(i)	$\text{Var}(2A-3B) = 4\text{Var}(A) + 9\text{Var}(B) - 12\text{Cov}(A,B)$ $\Rightarrow 18 = 36 + 54 - 12\text{Cov}(A,B)$ $\Rightarrow \text{Cov}(A,B) = 6$	M1 A1 A1 **3**	Correct formula**.** Allow one error Substitute relevant values CAO
(ii)	Since $\text{Cov}(A, B) \neq 0$, A and B are not independent	B1 ft **1** **(4)**	Must have a reason. ft $\text{Cov} \neq 0$
2(i)	$G'(t) = 8te^{4t^2}/e^4$ $E(X) = G'(1)$ $= 8$	M1A1 A1 **3**	M1 for ct^2/e^4
(ii)	EITHER: $G(t) = e^{-4}(1 + 4t^2 + \ldots)$ $P(X=2)$ = coefficient of $t^2 = 4e^{-4}$ or $4/e^4$ or 0.0733 OR $G''(t) = (8+64t^2)e^{4t^2-4}$ $P(X=2) = \frac{1}{2}G''(0) = 4e^{-4}$ or $4/e^4$ or 0.0733	M1A1 A1 **3** M1A1 A1 **(6)**	Expand in powers of t M1 for reasonable attempt at $M''(t)$
3(i)	Number of different rankings $^{11}C_5$ $=462$ For $R \leq 17$: $1+2+3+4+5 = 15$ $1+2+3+4+6=16$ $1+2+3+5+6=17$ $1+2+3+4+7=17$ $P(R \leq 17) = 4/462 = 2/231$ AG	M1 A1 B2 A1 **5**	Number of selections of 5 from 11 B1 for 2 or 3 correct
(ii)	$W = 17$ $P(W \leq 17) = \frac{2}{231}$ Smallest SL = $\frac{400}{231}$ %	M1 A1ft **2** **(7)**	 Allow $\frac{4}{231}$; ft $\frac{2}{231}$, but must be exact
4(i)	EITHER**:** (α) $M'(t) = n(1-2t)^{-½n-1}$ $E(Y) = M'(0)=n$ $M''(t) = n(n+2)(1-2t)^{-½n-2}$ $\text{Var}(Y) = n(n+2) - n^2 = 2n$ OR: $M(t) = 1 + nt + \frac{1}{2}n(n+2)t^2$ $E(Y) = n$ $\text{Var}(Y) = n(n+2) - n^2 = 2n$	M1 A1 A1 M1 A1 **5** M1A1A1 A1 A1 **5**	Correct form for M1 Ft similar $M'(t)$ $M''(0) - (M'(0))^2$
(ii)	$\text{MGF} = (1-2t)^{-30}$ χ^2 distribution with 60 d.f.	B1 B1 **2**	From $[(1-2t)^{-1/2}]^{60}$
(iii)	$E(S) = 60$, $\text{Var}(S) = 120$ Using CLT, Probability $=1 - \Phi(10/\sqrt{120})$ $= 0.181$	B1ft M1 A1 **3** **(10)**	From **(i)** Correct tail: allow cc

5(i)	Assumes salaries symmetrically distributed H_0: m(edian) = 19.5, H_1: m(edian)≠ 19.5 P = 867 (or 408) Using normal approximation $\mu = ¼ \times 50\times51$ (= 637.5) $\sigma^2 = 50\times51\times101/24$ (= 10731.25) $z = (a – 637.5)/\sqrt{10731.25}$ Use a = 866.5 = 2.211, or 2.215 or 2.220 (– from 408) Compare their z with 1.96 and reject H_0 There is sufficient evidence at the 5% SL that the median salary differs from £19 500	B1 B1 M1 A1 A1 M1 A1 A1 M1 A1 ft **10**	In context For both ; not μ ; accept words a=866.5, 867, 867.5 (or 408.5, 408, 407.5) Or p-value rounding to 0.026 or 0.027 Compare with 0.05 or equivalent ft z Or find critical region
(ii)	Use sign test when salary distribution is skewed	B1 **1** **(11)**	
6(i)	$\begin{array}{cc\|ccc} & & & N & \\ & & 0 & 1 & 2 \\ & 0 & 0 & c & 2c \\ R & 1 & 2c & 3c & 4c \\ & 2 & 4c & 5c & 6c \end{array}$ Total $27c = 1$ $c = \frac{1}{27}$	B1 M1 A1 **3**	 Calculate 9 probs in terms of c
(ii)	$9c/27c$ $= \frac{1}{3}$	M1 A1 ft **2**	Marginal probability AEF; ft c
(iii)	P($N + R > 2$) $= 15c/27c = \frac{5}{9}$	M1 A1 ft **2**	 AEF; ft c
(iv)	P(R=2) = $\frac{15}{27}$ P(N | R=2): $p_0 = \frac{4}{15}$, $p_1 = \frac{1}{3}$, $p_2 = \frac{2}{5}$ E(N |R=2)= $1\times\frac{1}{3}+2\times\frac{2}{5}$ $= \frac{17}{15}$	 M1 A1 ft A1 ft A1 **4**	 Using conditional probabilities One value; ft values in **(i)** All values Or 1.13
(v)	Eg P(N = 0 and R = 0) = 0 P(N=0)×P(R=0) = $\frac{6}{27}\times\frac{3}{27} \neq 0$ So N and R are not independent	M1 A1 **2** **(13)**	Or from conditional probs M0 from N=1 with R=1 or 2 All correct

7(i)	$\int_0^{2\theta} \frac{x^{n+1}}{2\theta^2}\,dx = \left[\frac{x^{n+2}}{2(n+2)\theta^2}\right]$ $= 2^{n+1}\theta^n/(n+2)$ $E(X) = 4\theta/3$	M1 A1 B1 ft **3**	Correct integral AEF B0 if not 'deduced'
(ii)	$Var(X) = 2\theta^2 - (4\theta/3)^2 = 2\theta^2/9$ $Var(X^2) = E(X^4) - (E(X))^2$ $= 16\theta^4/3 - 4\theta^4 = 4\theta^4/3$	M1A1ft M1A1ft **4**	ft **(i)** with no n ft **(i)** with no n
(iii)	$E(\sum X_i) = 3\times 4\theta/3$ $= 4\theta$ $T_1 = \frac{1}{4}\sum X_i$ $E(\sum X_i^2) = 3\times 2\theta^2$ $=6\theta^2$ $T_2 = (\sum X_i^{\,2})/27$	M1 A1 ft A1 ft M1 A1 ft A1 ft **6**	 ft with no n ft with no n or θ ft with no n ft with no n or θ
(iv)	$Var(T_2) = 1/27^2 \times 3 \times Var(X^2)$ $= 4\theta^4/729$	M1 A1 **2** **(15)**	 CAO
8(i)	$P(L\cap M) = P(L\mid M)P(M) = 0.12$ and $P(L) = P(M\cap L)/P(M\mid L)$ = 0.12/0.4 = 0.3 $P(L'\cup M') = P\left[(L\cap M)'\right]$ $=1-P(L\cap M)$ = 1 – 0.2 × 0.6 = 0.88	*M1* A1 B1 **3**	
(ii)	$P(N\mid L\cap M) = 0.3$ $\Rightarrow P(N\cap L\cap M) = 0.3\times 0.12$ =0.036 $P(L'\cup M'\cup N') = 1-0.036 = 0.964$	M1 A1 A1 **3** **[6]**	

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GCE

Mathematics

Advanced GCE **4736**

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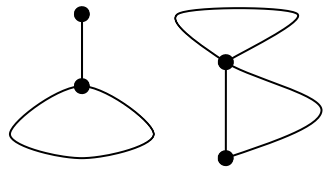
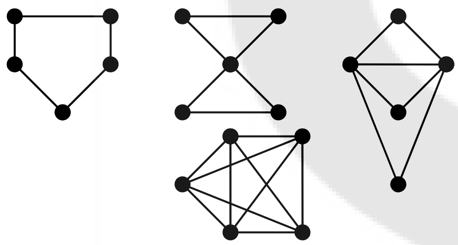
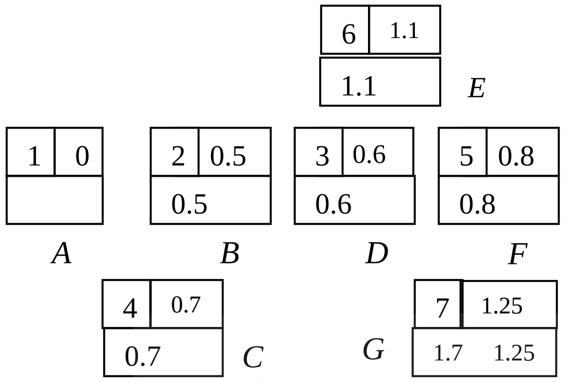
1(i) (a)	31 75 87 42 43 70 56 61 95 28 (may be shown vertically or as separate swaps) 9 comparisons and 8 swaps The smallest (final) mark, 28	M1 A1 B1 B1	**[4]**	28 moved to the end of the list, no other values moved Correct list at end of first pass (cao) 9 and 8 (written, not tallies) (cao) - if not specified, assume the larger value is comparisons (their) 28 or smallest/least or final/last/end If sorted into increasing order: 28 31 75 42 43 70 56 61 87 95 M0 A0, then 9 and 6 = B1 and (their) 95 or largest/greatest/biggest or final/last/end = B1
(b)	75 87 42 43 70 56 61 95 31 28	B1	**[1]**	Correct list at end of second pass If sorted into increasing order and already penalised in (i)(a) then condone here: 28 31 42 43 70 56 61 75 87 95
(c)	7 more passes	B1	**[1]**	7 (cao)
(ii)	31 28 75 87 42 43 70 56 61 95 75 31 28 87 42 43 70 56 61 95 1 comparison and 0 swaps in first pass 2 comparisons and 2 swaps in second pass	M1 A1 B1 B1	**[4]**	31 28 75 or 31 28 75 … Correct list, in full, at end of second pass Lists must be easily found, not picked out from working, if the candidate has labelled passes use them as labelled 1 and 0 (written)(cao) may appear next to list 2 and 2 (written)(cao) may appear next to list If sorted into increasing order: 28 31 75 … M0, A0, then 1 and 1 = B1; 1 and 0 = B1
(iii)	Bubble sort does not terminate early, since it takes 9 passes to get 95 to the front of the list, so it uses 9+8+…+1 or 45 comparisons Shuttle sort takes fewer than 1+2+…+9 comparisons, since, for example, in the fourth pass 42 will be compared with 28, 31 and 75 but not with 87.	B1 B1	**[2]**	Identifying that bubble sort <u>does not terminate early</u> (Just stating 9+8+…+1 or 45 = B0) Allow 'the largest number is at the end of the list' or '95 at end' A good explanation of why shuttle sort requires fewer comparisons <u>in this particular case</u> Do not accept 'because the list is not in reverse order'
(iv)	$20\times\left(\frac{50}{10}\right)^2$ = 500 seconds	M1 A1	**[2]**	Correct method 500 seconds or 8 mins 20 sec (without wrong working)

2(i)	Cannot have an odd number of odd nodes Odd vertices come in pairs	B1	**[1]**	Sum of orders must be even Sum of orders is 9 so 4.5 arcs (which is impossible)
(ii)	eg  Many other correct possibilities	M1 A1	 **[2]**	A diagram showing a graph with four vertices that is not connected and not simple Vertices have orders 1, 2, 3, 4
(iii)	The vertex of order 4 needs to connect to four other vertices, but there are only three other vertices available, so one vertex must be joined twice or the vertex of order 4 is connected to itself. Hence the graph cannot be simple	M1 A1	 **[2]**	Specifically identifying that the problem is with the vertex of order 4 Explaining why the graph cannot be simple (either reason) and stating that simple cannot be achieved Ignore any claims about whether or not the graph is connected
(iv) (a)	Each vertex of order 4 connects to each of the others, since graph is simple. Hence the other two vertices must have order (at least) 3. But Eulerian, so all must have order 4.	B1	**[1]**	Any reasonable explanation, but not just a diagram of a specific case 'the other two must be odd but they can't because Eulerian' is not enough Note: the graph has five vertices
(b)	Graph is Eulerian - so each vertex order is even; simple - so no vertex has order more than 4; and connected - so no vertex has order 0. Hence each vertex has order either 2 or 4. But cannot have 3 or 4 vertices of order 4. So must have 0, 1, 2 or 5 vertices of order 4. 	B1 M1 A1	 **[3]**	Explaining why there are only four such graphs Or list all the possibilities (eg 22222 42222 44222 44444) Any two correct (note: must be simply connected and Eulerian) All four correct and no extras (apart from topologically equivalent variations)

3(i)	$y \geq x$ $x \geq 0$ $y \leq 7 - \frac{2}{3}x$	M1 M1 A1	 **[3]**	Boundaries $y = x$ and $x = 0$ in any form (may be shown as an equality or an inequality with inequality sign wrong) Boundary $2x + 3y = 21$ in any form <u>All</u> inequalities correct (and any extras do not affect the feasible region)
(ii)	$(0, 7) \Rightarrow 42$ $(4.2, 4.2) \Rightarrow 29.4$ or $(\frac{21}{5}, \frac{21}{5}) \Rightarrow \frac{147}{5}$ At optimum, $x = 0$ and $y = 7$ $P_1 = 42$	M1 A1 A1	 **[3]**	Substantially correct attempt at testing vertices (at least one vertex apart from (0, 0)) or using a line of constant profit (may be implied) Accept (0, 7) identified (cao) 42 (stated) (cao) NOT deduced from earlier working, unless identified
(iii)	$(4.2, 4.2)$ $P_k = 4.2(k + 6)$ or $4.2k + 25.2$	B1 B1	 **[2]**	cao cao
(iv)	Compare $kx + 6y$ with boundary $2x + 3y$ or algebraically, $4.2(k + 6)$ with 42 or $-\frac{k}{6}$ with $-\frac{2}{3}$ $\Rightarrow k \leq 4$ $k \leq 4$ or $k < 4$ implies M1, A1	M1 A1	 **[2]**	Algebraically or using line, <u>or implied</u> (allow = here) Accept $k < 4$ No need to say that $k > 0$, but candidates may also say $k > 0$ or $k \geq 0$ Note: k is continuous, so answers such as 'k = 1, 2, 3, 4' or 'k = 1, 2, 3', with no other working, would get M1, A0

4(i)	 Route: *A – B – D – F - G*	M1 A1 B1 B1 B1	**[5]**	1.7 shown as a temporary label at *G* All temporary labels correct with no extras (may not have written temporary label when it becomes permanent) All permanent labels correct (cao) Order of labelling correct (cao) This route written down (not reversed) (cao)
(ii)	Route Inspection problem	B1	**[1]**	Accept Chinese postman Allow 'postman', 'postman route', but not just 'inspection'
(iii)	*CD* (*CBD*) = 0.3, *DG* (*DFG*) = 0.65, *CG* (*CBDFG*) = 0.95 *CD* (*CBD*) and *FG* = 0.75 or *CD* (*CBD*) and *EG* (*EFG*) = 1.05 Length = 3.7 + 0.5 + 0.3 + 0.75 = 5.25 km	M1 A1 M1 M1 A1	**[5]**	Any one of these seen (explicitly or as part of a calculation) All three of these seen (explicitly or as parts of calculations) Or either of these with *AB* to give 1.25 or 1.55 respectively Adding their 0.75 to 3.7 or their 0.75 to 3.7 + 0.5 + 0.3 (cao) units not needed 5.25 implies M1, M1 A1, irrespective of working
(iv)	*B – D - F – G – C – B* 1.9 km	B1 B1	**[2]**	cao 1.9 (cao) irrespective of method
(v)	[TREE] Vertices added in order *BDCF* or *BDFC* Arcs added in order *BD*, *BC*, *DF* or *BD*, *DF*, *BC* Two shortest arcs from *G* total 0.45 + 0.65 = 1.1 Lower bound = 0.5 + 1.1 = 1.6 km	B1 B1 M1 A1	**[4]**	Correct tree drawn A valid order of adding vertices or a valid order of adding arcs 0.45 and 0.65, or total 1.1 (may be implied from 1.6) 1.6 (cao) units not needed 1.6 implies M1, A1

Question	Answer	Marks	Total	Guidance
5(i)	$600x + 800y + 500z \le 5000$ $\Rightarrow 6x + 8y + 5z \le 50$	M1 A1		Correct inequality, allow < for M mark only Correct fully simplified form (cao)
	$120x + 80y + 120z \le 800$ $\Rightarrow 3x + 2y + 3z \le 20$	M1 A1	[4]	Correct inequality, allow < for M mark only Correct fully simplified form (cao)
	May use slack variables, provided they also specify slack variables non-negative eg $6x + 8y + 5z + t = 50$, $t \ge 0$ = M1, A1			If slack variable form used and fully simplified but without specifying that slack variables are non-negative, SC M1 A0 for each

(ii)

P	x	y	z	s	t	u	RHS
1	-100	-40	-120	0	0	0	0
0	12	20	15	1	0	0	60
0	6	8	5	0	1	0	50
0	3	2	3	0	0	1	20

M1 — Objective row correct and three slack variables used

A1 — Three constraint rows correct (ft (i), if reasonable)
Accept variations in order of rows and columns
Condone P column missing here

(ii)

$60 \div 15 = 4$, $50 \div 5 = 10$, $20 \div 3 = 6\frac{2}{3}$
Pivot on the 15 in the z column

B1 — Correct pivot choice from their z column

New row 2 = row 2 ÷ 15
New row 1 = row 1 + 120 × new row 2
New row 3 = row 3 – 5 × new row 2
New row 4 = row 4 – 3 × new row 2

M1 — Correct method for their pivot row seen (or implied from correct row in tableau if no attempt seen)
A1 — Correct method for their three other rows seen as a formula

P	x	y	z	s	t	u	RHS
1	-4	120	0	8	0	0	480
0	$\frac{4}{5}$	$1\frac{1}{3}$	1	$\frac{1}{15}$	0	0	4
0	2	$1\frac{1}{3}$	0	$-\frac{1}{3}$	1	0	30
0	$\frac{3}{5}$	-2	0	$-\frac{1}{5}$	0	1	8

M1 — Iterate to get a tableau with exactly four basis columns and non-negative entries in final column, in which the value of the objective has not decreased
A1 — Values in final column correct (follow through)

$4 \div \frac{4}{5} = 5$, $30 \div 2 = 15$, $8 \div \frac{3}{5} = 13\frac{1}{3}$
Pivot on the $\frac{4}{5}$ in the x column

B1 — Correct pivot choice for their second iteration

New row 2 = row 2 ÷ $\frac{4}{5}$
New row 1 = row 1 + 4 × new row 2
New row 3 = row 3 – 2 × new row 2
New row 4 = row 4 – $\frac{3}{5}$ × new row 2

M1 — Correct method for their pivot row seen (or implied from correct row in tableau if no attempt seen)
A1 — Correct method for their three other rows seen as a formula

P	x	y	z	s	t	u	RHS
1	0	$126\frac{2}{3}$	5	$8\frac{1}{3}$	0	0	500
0	1	$1\frac{2}{3}$	$1\frac{1}{4}$	$\frac{1}{12}$	0	0	5
0	0	-2	$-2\frac{1}{2}$	$-\frac{1}{2}$	1	0	20
0	0	-3	$-\frac{3}{4}$	$-\frac{1}{4}$	0	1	5

M1 — Iterate to get a tableau with exactly four basis columns and non-negative entries in final column, in which the value of the objective has not decreased
A1 — Values in final column correct (follow through)

	Make 5 litres of *fruit salad* only	B1	**[13]**	Interpretation of their final (non-negative) x, y and z, in context (need 'only' or equivalent; '5 *fruit salads*' is not enough) $x = 5$, $y = 0$, $z = 0$ gives B0
(iii)	$60 \div 12 = 5$, $50 \div 6 = 8\frac{1}{3}$, $20 \div 3 = 6\frac{2}{3}$ Pivot on the 12 in the x column	B1		Correct pivot choice from their x column
	New row 2 = row 2 ÷ 12	M1		Correct method for their pivot row (seen or implied from correct row in tableau)
	New row 1 = row 1 + 100 × new row 2	A1		Correct method for their objective row seen as a formula
	Showing that there are no negative entries in objective row	M1		Showing that there are no negative entries in objective row Or achieving a final tableau, in one iteration, with exactly four basis columns and non-negative entries in final column, in which the value of the objective has not decreased
	Saying that optimum has been achieved ('no negatives in top row')	A1	**[5]**	

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RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **4737**

Decision Mathematics 2

Mark Scheme for June 2010

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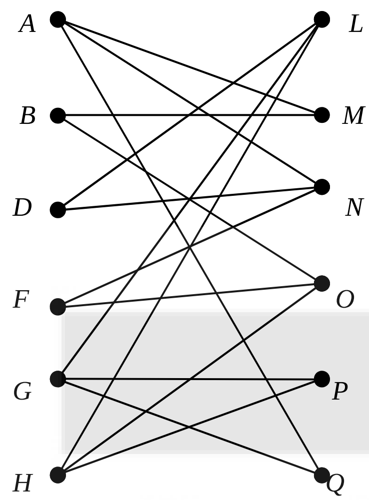
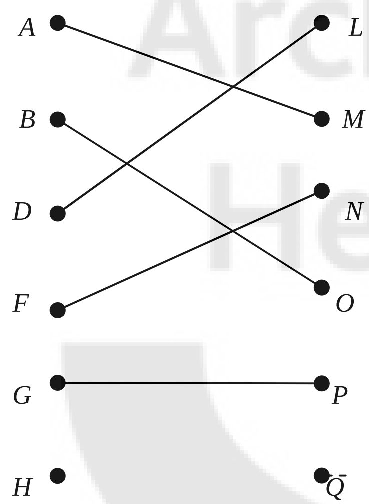
1	(i)		B1	A correct bipartite graph	[1]
	(ii)		B1	A second bipartite graph showing the incomplete matching correctly No augmentations made, even if in pencil. Ignore the addition of an X vertex though.	[1]
	(iii)	$H – P – G – Q$ Axe handle = Prof Mulberry $A = M$ Broomstick = Miss Olive $B = O$ Drainpipe = Mrs Lemon $D = L$ Fence post = Mr Nutmeg $F = N$ Golf club = Rev Quince $G = Q$ Hammer = Capt Peach $H = P$	B1 B1	This path in any reasonable form or in reverse. Accept X-H-P-G-Q Not any longer path from H to Q This complete matching written down (use initials of surnames if ambiguous, eg Rev Pineapple is interpreted as P = Capt Peach)	[2]
	(iv)	Axe handle = Rev Quince $A = Q$ Broomstick = Prof Mulberry $B = M$ Drainpipe = Mr Nutmeg $D = N$ Fence post = Miss Olive $F = O$ Golf club = Capt Peach $G = P$ Hammer = Mrs Lemon $H = L$	M1 A1	A different complete matching in any form A valid complete matching in which none of the suspects uses the same weapon as in their solution to (iii)	[2]
				Total =	**6**

2 (i)

	1 pm	2 pm	3 pm	4 pm	5 pm
R	7	6	8	3	9
S	5	0	4	4	4
T	6	3	7	5	7
W	4	2	6	2	7
Y	2	2	3	6	7

M1 Modify table by subtracting each entry from a constant value

A1 Correct table (ie this $\pm$ a constant throughout, with no negative values) **[2]**

Reduce rows

4	3	5	0	6
5	0	4	4	4
3	0	4	2	4
2	0	4	0	5
0	0	1	4	5

M1 Substantially correct attempt to reduce rows (at most 2 independent errors)

Reduce columns

4	3	4	0	2
5	0	3	4	0
3	0	3	2	0
2	0	3	0	1
0	0	0	4	1

M1 Substantially correct attempt to reduce columns (at most 2 independent errors)

A1 Their reduced cost matrix **[3]**

Cross out 0's using minimum no. of lines

4	3	4	0	2
5	0	3	4	0
3	0	3	2	0
2	0	3	0	1
0	0	0	4	1

Augment

2	3	2	0	2
3	0	1	4	0
1	0	1	2	0
0	0	1	0	1
0	2	0	6	3

M1 Substantially correct attempt at augmenting (at most 2 errors)

A1 Their matrix augmented correctly to reach a complete matching **[2]**

	1 pm	2 pm	3 pm	4 pm	5 pm
R	2	3	2	0	2
S	3	0	1	4	0
T	1	0	1	2	0
W	0	0	1	0	1
Y	0	2	0	6	3

Mrs Rowan = 4 pm or = 4 pm
Dr Silverbirch = 2 pm or = 5 pm
Mr Thorn = 5 pm or = 2 pm
Ms Willow = 1 pm or = 1 pm
Sgt Yew = 3 pm or = 3 pm

B1 First matching, cao

B1 Second matching, cao **[2]**

(ii) Mr Thorn

B1 Follow through their matchings (but not to S) **[1]**

Total = 10

3 (i)

Stage	State	Action	Working	Suboptimal minima
3	0	0	5	5
	1	0	4	4
	2	0	6	6
2	0	0	5 + 5 = 10	10
		1	6 + 4 = 10	10
	1	0	3 + 5 = 8	8
		1	5 + 4 = 9	
	2	1	3 + 4 = 7	7
		2	2 + 6 = 8	
1	0	0	2+10 = 12	
		1	3 + 8 = 11	11
	1	1	2 + 8 = 10	10
		2	3 + 7 = 10	10
	2	2	8 + 7 = 15	15
0	0	0	6 + 11 = 17	17
		1	8 + 10 = 18	
		2	3 + 15 = 18	

Minimum route = (0;0) - (1;0) - (2;1) - (3;0) – (4;0)
Weight = 17

Mark	Guidance	Total
B1	Structure of table correct (stage, state, action and 'working' columns)	
M1	Stage and state values correct	
A1	Action values correct	[3]
M1	Working column substantially correct for stage 2 (calcs or totals) (at most 1 error)	
A1	Suboptimal minima (10, 8, 7) correct for stage 2 (cao)	[2]
M1	Working column substantially correct for stage 1(at most 1 error)	
A1	Suboptimal minima (11, 10, 15) correct for stage 1 (cao)	[2]
B1	Correct route from (0; 0) to (4; 0)	[2]
B1	17 cao (written down, not just implied from table)	

(ii)

Start at the bottom of the table at (0; 0)
Optimum for stage 0 comes from action 0,
so (0; 0) connects to (1; 0)
Optimum for (1; 0) comes from action 1,
so (1; 0) connects to (2; 1)
Optimum for (2; 1) comes from action 0
so (2; 1) connects to (3; 0) and hence to (4; 0)

Mark	Guidance	Total
M1	Start at (0; 0), action 0 or value 11 (theirs), hence (1; 0)	
A1	(1; 0), action 1 (theirs), hence (2; 1) Clearly relating action to state for stage above	[2]

Total = **11**

<table>
<tr><td>4</td><td>(i)</td><td>In each game, whatever combination of strategies is chosen, the total number of points won is zero</td><td>B1</td><td>Points won by Euan equals points lost by Wai Mai, and vice versa, in every case</td><td>[1]</td></tr>
<tr><td></td><td>(ii)</td><td>-2</td><td>B1</td><td>Loses 2</td><td>[1]</td></tr>
<tr><td></td><td>(iii)</td><td>Z is dominated by Y

In <u>each</u> row she loses more by choosing Z than Y
-3 < 5, -4 < 3, -2 < 5 and 1 < 2 (or equivalent)</td><td>M1

A1</td><td>Idea of dominance by Y

Four valid comparisons <u>and</u> a convincing explanation (or equivalent in words)</td><td>[2]</td></tr>
<tr><td></td><td>(iv)</td><td>Wai Mai
<table>
<tr><td></td><td></td><td>X</td><td>Y</td><td>row min</td><td></td></tr>
<tr><td rowspan="4">Euan</td><td>A</td><td>2</td><td>-5</td><td>-5</td><td></td></tr>
<tr><td>B</td><td>-1</td><td>-3</td><td>-3</td><td></td></tr>
<tr><td>C</td><td>3</td><td>-5</td><td>-5</td><td></td></tr>
<tr><td>D</td><td>3</td><td>-2</td><td>-2</td><td>*</td></tr>
<tr><td>col max</td><td></td><td>3</td><td>-2</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>*</td><td></td><td></td></tr>
</table>

Play-safe for Euan is D
Play-safe for Wai Mai is Y

Game is stable,
since row maximin = col minimax, -2 = -2</td><td>M1

A1
A1

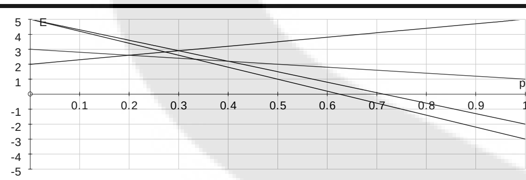
B1</td><td>Determining row minima and column maxima, or equivalent (may be implied from both D and Y stated)

D, stated (not just identified in table)
Y, stated (not just identified in table)

Stable, with a valid reason attempted (numerical or in words) (www)</td><td>[4]</td></tr>
<tr><td></td><td>(v)</td><td>A: $-2p + 5(1-p) = 5 - 7p$
B: $p + 3(1-p) = 3 - 2p$
C: $-3p + 5(1-p) = 5 - 8p$
D: $5p + 2(1-p) = 2 + 3p$

(note: leaving DX as 3 gives D: $2 - 5p$ = M1A0A0)</td><td>M1

A1

A1</td><td>Any one correct (or negative of correct), simplified or not
All four correct (or negative of correct) and simplified
All four correct and simplified</td><td>[3]</td></tr>
<tr><td></td><td>(vi)</td><td>

$2 + 3p = 3 - 2p$
$\Rightarrow p = 0.2$</td><td>M1

A1

M1
A1</td><td>Graph paper used with sensible scales
Their equations plotted correctly

Solving correct pair, or from graph
0.2, cao, from correct equations used (algebraically or from graph) (www)</td><td>[2]

[2]</td></tr>
<tr><td colspan="5" align="right">Total =</td><td>15</td></tr>
</table>

ANSWERED ON INSERT

5	**(i)**	21+36 +7 +18 = 82	M1 A1	Evidence of using the correct cut (eg 21 (± 23) + 36 + 7 + 18 seen) 82	**[2]**
	(ii)	At most 17 can leave *C* so there cannot be as much as 20 or 18 entering it At most 17 can enter *E* so there cannot be 7 + 18 = 25 leaving it Maximum that can flow in arc *HT* is 33 Flow along arc *HG* = 0	B1 B1 B1 B1	17 < both 20 and 18 (NOT 17 < 38) 17 < 7 + 18 33 0	**[2]** **[2]**
	(iii)	A diagram showing a flow of 58 in which amount in equals amount out at each vertex, apart from *S* and *T* Arcs *CE*, *FH* and *GT* are saturated and other arc capacities are not exceeded Cut *X* = {*S*, *A*, *B*, *C*, *D*, *F*, *G*}, *Y* = {*E*, *H*, *T*} Or cut through *GT*, *GH*, *FH*, *EF* and *CE*	M1 A1 B1	Assume that "blanks" mean 0 or full to capacity, provided consistent This cut presented in any form (accept it drawn on diagram)	**[3]**
	(iv)	Substantially correct attempt in which excess capacities and potential backflows marked correctly on arcs *CE*, *FH* and *GT* Their excess capacities and potential backflows marked correctly on arcs out of *S* and arcs into *T* and on *HG*	M1 A1	Assume that blanks mean 0 Accept <u>all</u> directions swapped Check directions on <u>*HG*</u> carefully If no flow in (iii), or ambiguous, then any valid flow > 0 labelled correctly gets M1, but must also be a flow of 58 to get A1	**[2]**
	(v)	Feasible route(s) written that send an additional 2 through system (or more on follow through) All route(s) valid with an additional 2 along *GH*	M1 A1	Routes must be written out properly eg route *SBFGHT* by 2	**[2]**
	(vi)	Their flow from part (iii) augmented by their routes in part (v) No more can flow across the cut *X* = {*S*, *C*}, *Y* = {*A*, *B*, *D*, *E*, *F*, *G*, *H*, *T*}	M1 A1	Follow through if possible Any reasonable explanation	**[2]**
				Total =	**15**

PARTS (i), (ii) AND (iii) ANSWERED ON INSERT

<table>
<tr><td>6</td><td>(i)</td><td>
<table>
<tr><th>Activity</th><th>Duration</th><th>Predecessors</th></tr>
<tr><td>A</td><td>6</td><td>-</td></tr>
<tr><td>B</td><td>5</td><td>-</td></tr>
<tr><td>C</td><td>3</td><td>A, B</td></tr>
<tr><td>D</td><td>9</td><td>A</td></tr>
<tr><td>E</td><td>4</td><td>A, B</td></tr>
<tr><td>F</td><td>2</td><td>A, B</td></tr>
<tr><td>G</td><td>2</td><td>E, H</td></tr>
<tr><td>H</td><td>3</td><td>C, F</td></tr>
<tr><td>I</td><td>5</td><td>D, G</td></tr>
<tr><td>J</td><td>6</td><td>E, H</td></tr>
<tr><td>K</td><td>10</td><td>C, F</td></tr>
<tr><td>L</td><td>4</td><td>I</td></tr>
<tr><td>M</td><td>12</td><td>I</td></tr>
<tr><td>N</td><td>6</td><td>J, K, L</td></tr>
</table>
</td><td>B1

M1

A1</td><td>Predecessors correct for A to F
(entries for A and B may be blank)

Substantially correct attempt at predecessors for other activities
(at most 2 errors)

Predecessors all correct for G to N</td><td>[3]</td></tr>
<tr><td></td><td>(ii)</td><td>Dummy is needed between $\boxed{2}$ and $\boxed{3}$ so that C, E and F follow both A and B but D follows A only

Dummy is needed between $\boxed{4}$ and $\boxed{5}$ so that C and F do not share both a common start and a common finish</td><td>B1

B1</td><td>D does not follow B
(D follows A only)

Identifying C and F appropriately</td><td>[2]</td></tr>
<tr><td></td><td>(iii)</td><td>
<table>
<tr><th>$\boxed{1}$</th><th>$\boxed{2}$</th><th>$\boxed{3}$</th><th>$\boxed{4}$</th><th>$\boxed{5}$</th><th>$\boxed{6}$</th><th>$\boxed{7}$</th><th>$\boxed{8}$</th><th>$\boxed{9}$</th><th>$\boxed{10}$</th></tr>
<tr><td>0</td><td>6</td><td>6</td><td>9</td><td>9</td><td>12</td><td>15</td><td>20</td><td>24</td><td>32</td></tr>
<tr><td>0</td><td>6</td><td>7</td><td>10</td><td>10</td><td>13</td><td>15</td><td>20</td><td>26</td><td>32</td></tr>
</table>

Minimum project completion time = 32 minutes
Critical activities: A, D, I and M</td><td>B1
M1

A1

B1
B1</td><td>Early event times correct, in table
Substantially correct backwards pass
(at most 2 errors in total)
Late event times correct, in table

32, cao
A, D, I, M and no others, cao</td><td>[5]</td></tr>
<tr><td></td><td>(iv)</td><td>Early event time at $\boxed{9}$ becomes the larger of 24 and $9+x$

Early event time at $\boxed{10}$ becomes the larger of 32 and $15+x$, which then also becomes the late event time at $\boxed{10}$

Late event time at $\boxed{9}$ then becomes 26 or $9+x$</td><td>M1
A1

M1

A1</td><td>$9+x$
Larger of 24 and $9+x$

Considering the event times at $\boxed{10}$

Correct consideration of 26 and $9+x$</td><td>[4]</td></tr>
<tr><td></td><td>(v)</td><td>$x = 17$</td><td>B1</td><td>17</td><td>[1]</td></tr>
<tr><td></td><td></td><td></td><td></td><td>Total =</td><td>15</td></tr>
</table>

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