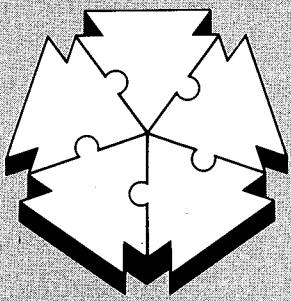


GCSE

Mathematics

Session: 1994
Type: Syllabus
Code: 1660



**Midland
Examining
Group**

**GENERAL CERTIFICATE OF
SECONDARY EDUCATION
NATIONAL CURRICULUM – KEY STAGE 4**

MATHEMATICS
(without Course Work)
Syllabus Code 1660

**Examination
Syllabus**

1660

GENERAL INFORMATION

Availability: This syllabus will be examined by the Midland Examining Group (MEG) in the Summer of the year shown on the cover.

In the Autumn of the year shown on the cover, only the central tier of the Scheme of Assessment will be available.

Certification: This subject will be shown on the GCSE certificate as

MATHEMATICS

Certificates will be issued by the Home Board on behalf of MEG.

Exclusions: In any one examination series, candidates entering for this subject may not in addition enter for any other MEG examination with the same certification title.

Entries: All candidates, including private candidates, must be entered by a Centre registered with MEG.

In order to enter candidates, a Centre must register with one of the MEG Boards (designated its Home Board). The Centre must make its entries for all MEG examinations through that Home Board.

All candidates must meet the full requirements of this syllabus and must therefore have their Course Work/Assessed Practical Work authenticated and assessed by an approved Centre.

Results: Results will be certificated as levels 4 - 10 of the National Curriculum ten level scale.

The relationship between the National Curriculum levels and GCSE grades certificated up to 1993 is shown below:

Grade	Level
A	10 9
B	8
C	7
D	6
E	
F	5
G	4

Enquiries: All enquiries about MEG or its examinations should be made to the Centre's Home Board.

Addresses and telephone numbers of the Boards in MEG are given on the back cover of this syllabus.

MATHEMATICS (without Course Work)

Syllabus Code 1660

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THE NATIONAL CURRICULUM AND THE GCSE

1. For the purposes of the National Curriculum, the Education Reform Act 1988 divides the period of compulsory schooling into four Key Stages. At the end of each Key Stage (ie at the ages of 7, 11, 14 and 16) attainments in National Curriculum subjects are to be tested and reported.
2. The GCSE is the main means of assessment at the end of Key Stage 4, but is not confined to candidates who are at that Stage. In subjects where National Curriculum Orders apply to Key Stage 4, GCSE subject criteria and syllabuses have been brought into line with the requirements of the Orders.
3. Outcomes of GCSE assessments in all subjects will in future be reported in **Levels** instead of grades. The relationship between the former GCSE grades and the National Curriculum Levels of Attainment is given in the front of this booklet.
4. The knowledge, skills and understanding which candidates are expected to have acquired in a subject at the end of each Key Stage are identified under headings known as **Attainment Targets**. The number of these varies according to the subject.
5. In each Attainment Target the Levels are described in one or more **Statements of Attainment**.
6. The National Curriculum Orders also contain **Programmes of Study** which describe the essential ground to be covered to enable the Attainment Targets to be met.
7. Attainment in the GCSE will be reported both as Levels for each Attainment Target and as a single Level for the subject.

1. INTRODUCTION

The GCSE is designed to assess candidates whose attainment at the end of Key Stage 4 falls within the range of levels 4-10 on the National Curriculum ten-level scale.

This GCSE syllabus has been designed to measure achievement in Mathematics in the National Curriculum (1991) and has been devised to be consistent with the requirements of the General and Subject Criteria. The Scheme of Assessment will allow attainment to be reported on the National Curriculum ten-level scale in respect of the whole subject and its five Attainment Targets, namely Using and Applying Mathematics, Number, Algebra, Shape and Space, and Handling Data.

The syllabus has been designed so that Centres may successfully use a wide range of teaching materials. It thus promotes and encourages flexibility and variety of teaching and learning styles employed in the classroom and will facilitate the effective and efficient use of resources.

Assessment is provided by terminal examination papers only and has a clear structure. Simplicity of implementation has been borne in mind in the design of the syllabus so that demands on both teachers and candidates are manageable.

The syllabus is designed to assess positive achievement at every level of ability specified for the GCSE. Candidates will be assessed within a range of levels of attainment so that they can show what they know, understand and can do.

The syllabus will encourage and support the provision and development of worthwhile and interesting courses, and in particular work of a cross curricular nature is fully encouraged.

2. AIMS

To encourage candidates to develop:

1. a positive attitude to mathematics, including confidence, enjoyment and perseverance;
2. an appreciation of the place and use of mathematics in society, including historical and cultural influences;
3. an ability to think and communicate mathematically - precisely, logically and creatively;
4. a willingness and ability to work both independently and co-operatively;
5. the ability to apply mathematical knowledge and understanding to solve problems and to present solutions clearly, interpreting and checking results;
6. an appreciation of pattern and relationships in mathematics;
7. the ability to classify, generalise and justify or prove statements;
8. an understanding of the interdependence of different branches of mathematics;
9. the knowledge, skills and understanding needed to apply a range of mathematical concepts to situations which may arise in their own lives;
10. the skills to explore mathematical ideas;
11. the ability to use mathematics across the curriculum;
12. a firm foundation for further study.

This syllabus is based on the aims listed above. The contribution of these aims to the quality of learning is likely to be considerable, although it may not be feasible or desirable to link all of the aims directly to the Assessment Objectives.

3. ASSESSMENT OBJECTIVES

The Attainment Targets and associated Statements of Attainment of the Statutory Order for National Curriculum Mathematics constitute the Assessment Objectives of the syllabus (see Appendix). These Assessment Objectives also reflect the statutory Programmes of Study.

4. SCHEME OF ASSESSMENT

The Scheme of Assessment consists of terminal examination papers only.

Candidates will enter at only one of three tiers, namely Basic, Central or Further, as set out below.

Candidates must enter for either Components 1 and 4 (Basic Tier) or Components 2 and 5 (Central Tier) or Components 3 and 6 (Further Tier).

Basic Tier

Component 1:	Short-and-longer answer terminal examination Paper 1
Component 4:	Structured/extended answer terminal examination Paper 4

Papers 1 and 4 are each of 2 hours' duration.

Central Tier

Component 2:	Short-and-longer answer terminal examination Paper 2
Component 5:	Structured/extended answer terminal examination Paper 5

Papers 2 and 5 are each of 2 hours 30 minutes' duration.

Further Tier

Component 3:	Short-and-longer answer terminal examination Paper 3
Component 6:	Structured/extended answer terminal examination Paper 6

Papers 3 and 6 are each of 2 hours 30 minutes' duration.

At each tier of entry, the two specified terminal examination papers each comprise 50% of the assessment and assess, with a 10% weighting for each, Attainment Targets 1, 2, 3, 4 and 5.

The assessment of each of the five Attainment Targets forms 20% of the total assessment.

The Attainment Targets 1-5 are assessed at each tier of entry by the two specified terminal examination papers. Each paper will be divided into Sections A and B. Section A will consist of questions assessing Attainment Targets 2-5 while section B will consist of questions assessing Attainment Target 1. Section A of each paper forms the corresponding paper of syllabus 1661 Mathematics (with Course Work). Section B of the two specified papers at each tier of entry is offered as an alternative to the Course Work component of syllabus 1661 for the assessment of Attainment Target 1.

The short-and-longer answer papers will be timetabled in one session and the structured/extended answer papers in another session. All candidates will take the short-and-longer answer paper first and only two timetable sessions will be occupied.

Differentiation

For the terminal examination papers, candidates will enter at one of three tiers, namely Basic, Central or Further. At each tier the terminal examination papers will be designed to assess those candidates whose ability corresponds to the National Curriculum levels called **target levels** in the table below.

TIER	TERMINAL EXAMINATION PAPERS	TARGET LEVELS	LEVELS AVAILABLE
Basic	1, 4	4, 5, 6	3, 4, 5, 6, *
Central	2, 5	7, 8	5, 6, 7, 8, *
Further	3, 6	9, 10	7, 8, 9, 10

Level 3 will not be reported on GCSE certificates.

* In exceptional cases, level 7 may be awarded to candidates entered for the Basic Tier, and level 9 may be awarded to candidates entered for the Central Tier.

National Curriculum Attainment Targets in Mathematics are arranged by levels of attainment in increasing order of difficulty. Centres should enter for each tier only those candidates expected to achieve one of the **target levels** for that tier, as the majority of the marks available for each paper will be directed towards the target levels. The remaining levels available at a tier provide a ‘safety net’ for those candidates not performing up to expectations in the terminal examination. In this way candidates will be examined at tiers at which they can demonstrate achievement and provide positive evidence of attainment.

In accordance with the General Criteria, the outcome W will be recorded where a candidate's performance provides insufficient evidence for the award of a level.

Recommended tiers of entry are given in the table below.

For ease of comparison, the corresponding lettered grades previously used for GCSE certification are shown in brackets.

CANDIDATE'S EXPECTED ATTAINMENT LEVEL	RECOMMENDED TIER OF ENTRY
4 (G), 5 (F) or 6 (E/D)	Basic
7 (C/D) or 8 (B)	Central
9 (A) or 10	Further

Assessment Grid

The grid below summarises the structure of the Scheme of Assessment.

COMPONENT NUMBER	COMPONENT TITLE	TARGET LEVELS	LEVELS AVAILABLE	ATTAINMENT TARGETS ASSESSED	DURATION	PERCENTAGE WEIGHTING
<i>Short-and-longer answer paper</i>						
1. Basic	Paper 1	4, 5, 6	3, 4, 5, 6, *	1, 2, 3, 4, 5	2 hrs	50% (i.e. 10% for each of Attainment Targets 1, 2, 3, 4, 5)
2. Central	Paper 2	7, 8	5, 6, 7, 8, *	1, 2, 3, 4, 5	2 hrs 30 min	50% (i.e. 10% for each of Attainment Targets 1, 2, 3, 4, 5)
3. Further	Paper 3	9, 10	7, 8, 9, 10	1, 2, 3, 4, 5	2 hrs 30 min	50% (i.e. 10% for each of Attainment Targets 1, 2, 3, 4, 5)
<i>Structured/extended answer paper</i>						
4. Basic	Paper 4	4, 5, 6	3, 4, 5, 6, *	1, 2, 3, 4, 5	2 hrs	50% (i.e. 10% for each of Attainment Targets 1, 2, 3, 4, 5)
5. Central	Paper 5	7, 8	5, 6, 7, 8, *	1, 2, 3, 4, 5	2 hrs 30 min	50% (i.e. 10% for each of Attainment Targets 1, 2, 3, 4, 5)
6. Further	Paper 6	9, 10	7, 8, 9, 10	1, 2, 3, 4, 5	2 hrs 30 min	50% (i.e. 10% for each of Attainment Targets 1, 2, 3, 4, 5)

Level 3 will not be reported on GCSE certificates.

* In exceptional cases, level 7 may be awarded to candidates entered for the Basic Tier, and level 9 may be awarded to candidates entered for the Central Tier.

Assessment Components

Terminal Examination Papers (100% of the assessment)

On each paper, candidates will be required to attempt all questions.

Each paper will cover the levels shown below of Attainment Targets 1 (Using and Applying Mathematics), 2 (Number), 3 (Algebra), 4 (Shape and Space) and 5 (Handling Data), with an equal number of marks being allocated to each of these Attainment Targets. Sampling of Statements of Attainment by the papers will be in accordance with the requirements of the General Criteria. Each paper will be divided into Sections A and B. Section A will consist of questions assessing Attainment Targets 2-5 while Section B will consist of questions assessing Attainment Target 1. In Section A questions may be set which contribute to the assessment of more than one Attainment Target, but the mark scheme will enable a candidate's performance in each Attainment Target to be identified.

For Basic Tier Papers 1 and 4, the responses to each paper will be written on the question paper. For Central Tier Papers 2 and 5 and Further Tier Papers 3 and 6, the responses to Section A will be written on the question paper and the responses to Section B will be written on separate answer paper.

Candidates will be expected to carry out practical work, make appropriate use of computers and respond orally during the course. Where appropriate, oral exchanges should involve a candidate carrying out mental calculations. Questions will be set on the terminal examination papers which draw on the experience of these activities.

The papers will be designed to be as free as possible from bias and prejudice.

Basic Tier

Papers 1 and 4 will each cover levels 3, 4, 5, 6 (Target levels 4, 5, 6).

- Paper 1 will consist of short and longer answer questions. Some of the questions or parts of questions will be common to Central Tier Paper 2.
- Paper 4 will consist of structured/extended answer questions. The degree of structure within the questions will be appropriate to the Target levels 4, 5, 6.

Central Tier

Papers 2 and 5 will each cover levels 5, 6, 7, 8 (Target levels 7, 8).

- Paper 2 will consist of short and longer answer questions. Some of the questions or parts of questions will be common to Basic Tier Paper 1 and some of the questions or parts of questions will be common to Further Tier Paper 3.
- Paper 5 will consist of structured/extended answer questions. The degree of structure within the questions will be appropriate to the Target levels 7, 8.

Further Tier

Papers 3 and 6 will each cover levels 7, 8, 9, 10 (Target levels 9, 10).

- Paper 3 will consist of short and longer answer questions. Some of the questions or parts of questions will be common to Central Tier Paper 2.
- Paper 6 will consist of structured/extended answer questions. The degree of structure within the questions will be appropriate to the Target levels 9, 10. There will thus be an emphasis on extended answer questions.

Formulae and the use of calculators and computers

In all papers candidates will be expected to use an electronic calculator with at least the four functions $+$, $-$, \times , \div , a square root key and single memory.

At the Central and Further tiers, candidates will be expected to use calculators with trigonometric functions and an x^y key.

At all tiers, relevant formulae will be provided on the inside front covers of the question papers.

Aggregation

Aggregation of Terminal Examination marks to determine Attainment Target and overall subject levels will be according to SEAC General Criteria and the Joint Council Standing Agreement on Principles for Assessment, Moderation, Awarding and Reporting in Foundation Subjects: GCSE.

5. ATTAINMENT LEVEL DESCRIPTIONS

The Statements of Attainment and Programmes of Study at each attainment level contained in the Statutory Order for National Curriculum Mathematics constitute the attainment level descriptions for the syllabus (see Appendix).

APPENDIX

The five Attainment Targets are

ATTAINMENT TARGET 1: USING AND APPLYING MATHEMATICS

Pupils should choose and make use of knowledge, skills and understanding outlined in the Programmes of Study in practical tasks, in real-life problems and to investigate within mathematics itself. Pupils would be expected to use with confidence the appropriate mathematical content specified in the programmes of study relating to the other attainment targets.

ATTAINMENT TARGET 2: NUMBER

Pupils should understand and use number, including estimation and approximation, interpreting results and checking for reasonableness.

ATTAINMENT TARGET 3: ALGEBRA

Pupils should recognise and use symbolic and graphical representation to express relationships.

ATTAINMENT TARGET 4: SHAPE AND SPACE

Pupils should recognise and use the properties of two dimensional (2-D) and three dimensional (3-D) shapes and use measurement, location and transformation in the study of space.

ATTAINMENT TARGET 5: HANDLING DATA

Pupils should collect, process and interpret data and should understand, estimate and use probabilities.

The five Attainment Targets for levels 3-10 are listed on the following pages.

The Statements of Attainment are shown in bold type.

The Programmes of Study are shown in plain type.

Examples are shown in italic type.

Attainment Targets 2-5 are assessed by Section A of the terminal examination papers. These Attainment Targets are listed side-by-side so that information at a level or at a tier of entry can be easily accessed.

Attainment Target 1 is assessed by Section B of the terminal examination papers. This Attainment Target is listed separately.

**BASIC
TIER**

**ATTAINMENT TARGET 2 NUMBER
ATTAINMENT TARGET 3 ALGEBRA**

Pupils should be able to:
 Pupils should engage in activities which involve:
Pupils could:

(a) **Read, write and order numbers up to 1000.**
 Reading, writing and ordering numbers to at least 1000, and using the knowledge that the position of a digit indicates its value.

Learning and using addition and subtraction facts to 20 (including zero).
Use dice to generate 3 digits; make as many 3-digit numbers as you can; read them out and order them.

Explain that four hundred and two is written 402 and why neither 42 nor 4002 is correct.

(b) **Demonstrate that they know and can use multiplication tables.**
 Learning and using multiplication facts up to 5x5 and all those in the 2, 5 and 10 multiplication tables.

Give at least three possibilities for \blacksquare and \blacktriangle , given $\blacksquare \times \blacktriangle = 20$.

Explain that if tickets cost £4 each only four can be bought with £18.

Pupils should be able to:
 Pupils should engage in activities which involve:
Pupils could:

(a) **Use pattern in number when doing mental calculations.**
 Developing a variety of strategies to perform mental calculations using number patterns and equivalent forms of 2-digit numbers.

Explaining number patterns and predicting subsequent numbers.
 Recognising whole numbers divisible by 2, 5 or 10.
Continue: 5, 10, 15, 20 ...

Continue:
 $4 + 10 = 14$
 $14 + 10 = 24$
 $24 + 10 = 34$
 \dots

Explore patterns in art or PE.

(b) **Use inverse operations in a simple context.**
 Dealing with inputs to and outputs from simple function machines.

Use doubling and halving, adding and subtracting, FORWARD and BACKWARD (in LOGO), etc, as inverse operations.

Pupils should be able to:
 Pupils should engage in activities which involve:
Pupils could:

(a) **Sort shapes using mathematical criteria and give reasons.**
 Sorting 2-D and 3-D shapes and giving reasons for each method of sorting.

Sort shapes with a square corner, shapes with curved edges and shapes with equal sides or faces, giving appropriate explanations.

(b) **Recognise reflective symmetry.**
 Recognising (reflective) symmetry in a variety of shapes in two and three dimensions.

Explore patterns from a variety of world cultures, eg Islamic, Japanese. Study shapes and identify some lines and planes of symmetry.

(c) **Use the eight points of the compass to show direction.**
 Using and understanding compass bearings and terms 'clockwise' and 'anticlockwise'. *Describe wind direction from a weather vane.*

Describe locations of places in the neighbourhood of school, using compass points.

Pupils should be able to:
 Pupils should engage in activities which involve:
Pupils could:

(a) **Access information in a simple database.**
 Extracting specific pieces of information from tables and lists.

Entering and accessing information in a simple database, eg card database. Entering data into a simple computer database and using it to find answers to simple questions.

Read off a value from a table; find the cost of an item in a mail order catalogue; compare prices of similar items. Handle weather statistics or personal data, such as height, date of birth or age. Find information from bus or train timetables as part of planning a journey.

LEVEL 3

Find $27 + 31$ by:
 $27 + 31 = 20 + 7 + 30 + 1$
 $= 50 + 8$

(b) **Use inverse operations in a simple context.**
 Dealing with inputs to and outputs from simple function machines.

Use doubling and halving, adding and subtracting, FORWARD and BACKWARD (in LOGO), etc, as inverse operations.

**ATTAINMENT TARGET 4 SHAPE AND SPACE
ATTAINMENT TARGET 5 HANDLING DATA**

Pupils should be able to:
 Pupils should engage in activities which involve:
Pupils could:

(a) **Access information in a simple database.**
 Extracting specific pieces of information from tables and lists.

Entering and accessing information in a simple database, eg card database. Entering data into a simple computer database and using it to find answers to simple questions.

Read off a value from a table; find the cost of an item in a mail order catalogue; compare prices of similar items. Handle weather statistics or personal data, such as height, date of birth or age. Find information from bus or train timetables as part of planning a journey.

**BASIC
TIER**

**ATTAINMENT TARGET 2
NUMBER**

- (c) Solve problems involving multiplication or division. Solving problems involving multiplication or division of whole numbers or money, using a calculator where necessary.
- Understanding remainders in the context of calculation and knowing whether to round up or down.

Find the cost of four calculators at £2.45 each.

Find out how many ways seating can be arranged for 25 children in rows of not more than 10. Which is the best way?

- (d) Make estimates based on familiar units of measurement checking results.

Making estimates based on familiar units. Recognising that the first digit is the most important in indicating the size of a number and approximating to the nearest 10 or 100.

Estimate the height of a door in metres.

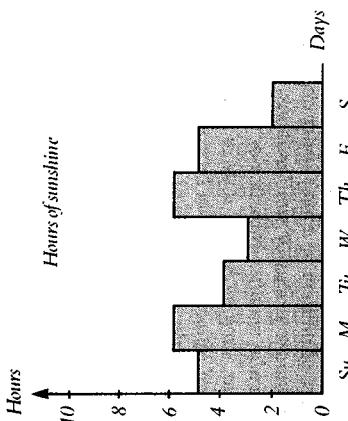
Estimate in centimetres the size of paper needed to design and make a greetings card.

Estimate the time it will take to clear away the PE equipment/art material.

LEVEL 3

**ATTAINMENT TARGET 5
HANDLING DATA**

- (b) Construct and interpret statistical diagrams.
- Constructing and interpreting bar charts, and graphs (pictograms) where the symbol represents a group of units.
- Decide what this graph tells you:*



Decide the number of raffle tickets sold by each class:

<input type="checkbox"/>	= 20	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>
Class A		<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>			
Class B		<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>			
Class C		<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>			
Class D		<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>			

[] = less than 20

**BASIC
TIER**

**ATTAINMENT TARGET 2
NUMBER**

- (e) Interpret a range of numbers in the context of measurement or money.
Using decimal notation in recording money.

Recognising negative whole numbers in familiar contexts, eg a temperature scale, a number line, a calculator display.
Using a wider range of metric units of length, capacity, weight and standard units of time.

Choosing and using appropriate units and instruments; interpreting numbers on a range of measuring instruments, with appropriate accuracy.

Read digital clocks correctly and analogue clocks to the nearest labelled division.
Read a thermometer, a stopwatch, a ruler, the dial on kitchen scales.

Appreciate that three £1 coins plus six 1p coins is written as £3.06 and that 3.6 on a calculator means £3.60 in the context of money.

LEVEL 3

**ATTAINMENT TARGET 5
HANDLING DATA**

- (c) Use appropriate language to justify decisions when placing events in order of 'likelihood'. Placing events in order of 'likelihood' and using appropriate words to identify the chance.

Understanding and using the idea of 'events' and saying whether events are more or less likely than this.
Distinguishing between 'fair' and 'unfair'.

Explain why the following game is unfair:
A bag contains 10 yellow and 1 red cube.
The rules of a game are:

- Player A is yellow; player B is red;
- the first player to pick his or her own colour out of the bag is the winner.

Decide, for each of these statements, if they are 'very likely', 'likely', 'unlikely', or 'very unlikely':
Ten people in my class will be away tomorrow;

'It will snow in the next half hour';
'My favourite television star will visit my school';
'We will do painting today'.

**BASIC
TIER**

ATTAINMENT TARGET 2 ATTAINMENT TARGET 3

ATTAINMENT TARGET 4 ATTAINMENT TARGET 5

Pupils should be able to:
Pupils should engage in activities which involve:
Pupils could:

- (a) Solve problems without the aid of a calculator considering the reasonableness of the answer.
Reading, writing and ordering whole numbers.
Learning multiplication facts up to 10x10 and using them in multiplication and division problems.
Adding and subtracting mentally two 2-digit numbers.
Adding mentally several single-digit numbers.
Adding and subtracting two 3-digit numbers, without a calculator.
Multiplying and dividing 2-digit numbers by a single-digit number, without a calculator.
Estimating and approximating to check the validity of addition and subtraction calculations.
Solving addition and subtraction problems using numbers with no more than 2 decimal places, and multiplication and division problems starting with whole numbers.

LEVEL 4
Find the total number of pupils in a school with 135 infants and 224 juniors and appreciate that the correct result must be over 300.
Work out mentally how much an object weighing 75kg is heavier than one weighing 48kg.
Understand the patterns in a place value system by examining ancient number systems from various parts of the world.

Pupils should be able to:
Pupils should engage in activities which involve:
Pupils could:

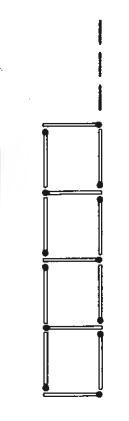
- (a) Make general statements about patterns.
Generalising, mainly in words, patterns which arise in various situations, eg symmetry of results, 'multiple', 'factor', 'square'.
Applying strategies such as doubling and halving to explore properties of numbers.
Recognising that multiplication and division are inverse operations and using this to check calculations.
Dealing with inputs and outputs from simple function machines.
Understand the patterns in addition and multiplication tables, including symmetry of results and relationships between multiplication by 2, 4 and 8 etc.

LEVEL 4
$$\begin{array}{r} x \\ \times 2 \\ \hline 8 \\ 12 \\ 14 \\ \hline 24 \end{array}$$

$$\begin{array}{r} x \\ \times 3 \\ \hline 12 \\ 18 \\ 21 \\ \hline 63 \end{array}$$

$$\begin{array}{r} x \\ \times 5 \\ \hline 20 \\ 30 \\ 35 \\ \hline 175 \end{array}$$

Construct matchstick squares, using an appropriate number of matchsticks, to make 1, 2, 3, 4, ... squares, and generalise the pattern.



Pupils should be able to:
Pupils should engage in activities which involve:
Pupils could:

- (a) Construct 2-D or 3-D shapes and know associated language.
Constructing simple 2-D and 3-D shapes from given information and knowing associated language.
Reflecting simple shapes in a mirror line.
Understanding congruence of simple shapes.
Understanding and using language associated with angle.
Construct rectangles, circles, nets for cubes, pyramids and prisms.
Know 'acute', 'obtuse', 'reflex', 'parallel', 'perpendicular', 'vertical', 'horizontal', etc.
Design and make a container for an awkwardly shaped object, eg 'Santa's lost boot'.

(b) Specify location.
Specifying location by means of coordinates in the first quadrant and by means of angle and distance.
Locate features on an ordnance survey map given their grid references.
Use turtle graphics instructions for distances and direction.
(c) Recognise rotational symmetry.
Recognising rotational symmetry.
Confirm the rotational symmetry of shapes using tracing paper.

Pupils should be able to:
Pupils should engage in activities which involve:
Pupils could:

- (a) Interrogate and interpret data in a computer database.
Inserting, interrogating and interpreting data in a computer database.
Interrogate a simple computer database to find plants suitable for creating a garden border which flowers blue and white in the summer months.
(b) Conduct a survey on an issue of their choice.
Specifying an issue for which data are needed.
Collecting, grouping and ordering discrete data using tallying methods and creating a frequency table for grouped data.
Understanding and using the median and mode in everyday contexts.
Constructing and interpreting bar-line and line graphs and frequency diagrams with suitable class intervals for discrete variables.
Creating a decision tree-diagram with questions to sort and identify a collection of objects.
Find and record the number of pupils born in each month of the year; produce a chart for display.
Conduct a survey across the school to find which five events would be most popular for a School Sports Day. Communicate the results in a variety of ways.

**BASIC
TIER**

**ATTAINMENT TARGET 2
NUMBER**

- (b) Demonstrate an understanding of the relationship between place values in whole numbers. Understanding and using the effect of multiplying whole numbers by 10 or 100.

Understanding and using the relationship between place values in whole numbers.

Explain why 5000 is 5 thousands or 50 hundreds or 5000 ones.

- c) Use fractions, decimals or percentages as appropriate to describe situations.

Recognising and understanding simple fractions in everyday use. Using, with understanding, decimal notation to 2 decimal places in the context of measurement, appreciating the continuous nature of measurement. Recognising and understanding simple percentages.

Estimate $\frac{1}{3}$ of a pint of milk or $\frac{3}{4}$ of the length of a piece of wood.

Read scales marked in hundredths and numbered in tenths (1.89m).

Know that 7 books out of a total of 14 books represents 50%.

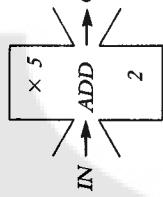
Find $\frac{3}{4}$ of £1, £7, £10.

**ATTAINMENT TARGET 3
ALGEBRA**

- (b) Use simple formulae expressed in words.

Understanding and using simple formulae or equations expressed in words.

Given that this machine multiplies all numbers by 5 then adds 2,



complete the table:

IN	OUT
2	12
3	?
?	37

Solve a problem such as: 'If I double a number then add 1, and the result is 49, what is the number?'

- (c) Use co-ordinates in the first quadrant.

Learning the conventions of the coordinate representation of points; working with co-ordinates in the first quadrant.

Draw diagrams by plotting points.

Create shapes by using turtle geometry in an appropriate computer language.

Draw graphs as required in other areas of the Programmes of Study and across the curriculum.

**ATTAINMENT TARGET 4
SHAPE AND SPACE**

- (d) Find perimeters, areas or volumes.

Finding perimeters of simple shapes. Finding areas by counting squares, and volumes by counting cubes.

Identify different rectangles with the same perimeter.

Compare the areas of leaves using a transparent square grid.

Work out the approximate volumes of small boxes.

Work out how many different rectangles can be made from 24 tiles. What is their area and perimeter?

Understanding and using the probability scale from 0 to 1.

Listing all the possible outcomes of an event.

Estimate, with reasons, the likelihood of rain tomorrow.

**ATTAINMENT TARGET 5
HANDLING DATA**

- (c) Use the mean and range of a set of data.

Understanding, calculating and using the mean and range of a set of data.

Calculate the mean and range to compare the scoring records of two hockey teams which have played different numbers of games.

- (d) Estimate and justify the probability of an event.

Giving and justifying subjective estimates of probabilities.

Understanding and using the probability scale from 0 to 1.

Listing all the possible outcomes of an event.

Estimate, with reasons, the likelihood of rain tomorrow.

LEVEL 4

**BASIC
TIER**

**ATTAINMENT TARGET 2
NUMBER**

- (d) Solve number problems with the aid of a calculator, interpreting the display.

Reading calculator displays to the nearest whole number and knowing how to interpret results which have rounding errors. Solving addition, subtraction, multiplication and division problems using numbers with no more than 2 decimal places.

Find the thickness of one page by measuring 100 pages of a book, to the nearest millimetre, and use this information to find the thickness of 246 pages.

Find out how many 47-seater coaches will be needed for a school trip for a party of 352.

Discover what length of wood will be left if three pieces measuring 2m 92cm, 3m 7cm, and 3m 21cm are cut from a piece of wood with a total length of 12m.

- (e) Make sensible estimates of a range of measures in relation to everyday objects.

Making sensible estimates of a range of measures in relation to everyday objects.

Understanding the relationship between the units of length, 'weight', capacity, time.

Estimate the length of a car, the capacity of a teacup, the 'weight' of a school bag.

Estimate the time taken to complete a task.

LEVEL 4

BASIC TIER	ATTAINMENT TARGET 2 NUMBER	ATTAINMENT TARGET 3 ALGEBRA	ATTAINMENT TARGET 4 SHAPE AND SPACE	ATTAINMENT TARGET 5 HANDLING DATA
AND TIER	Pupils should be able to: Pupils should engage in activities which involve: <i>Pupils could:</i>	Pupils should be able to: Pupils should engage in activities which involve: <i>Pupils could:</i>	Pupils should be able to: Pupils should engage in activities which involve: <i>Pupils could:</i>	Pupils should be able to: Pupils should engage in activities which involve: <i>Pupils could:</i>
CENTRAL TIER	<p>(a) Use an appropriate non-calculator method to multiply or divide two numbers.</p> <p>Understanding and using non-calculator methods by which a 3-digit number is multiplied by a 2-digit number and a 3-digit number is divided by a 2-digit number.</p> <p>Multiplying and dividing mentally single-digit numbers of powers of 10 with whole number answers.</p> <p>Use any pencil and paper method to find the number of coaches needed to take 165 Year 7 pupils on an outing if each coach has 42 seats.</p> <p>Calculate mentally 70×500, and $800 \div 20$.</p>	<p>(a) Follow instructions to generate sequences.</p> <p>Generating sequences.</p> <p>Recognising patterns in numbers through spatial arrangements.</p> <p>Understanding and using terms such as 'prime', 'cube', 'square root' and 'cube root'.</p> <p>Recognising patterns in equivalent fractions.</p> <p>Follow instructions to find all the prime numbers between 0 and 100.</p> <p>Understand the program:</p> <pre>10 FOR NUMBER = 1 TO 10 20 PRINT NUMBER * NUMBER 30 NEXT NUMBER 40 END.</pre>	<p>(a) Use accurate measurement and drawing in constructing 3-D models.</p> <p>Measuring and drawing angles to the nearest degree.</p> <p><i>Construct prisms.</i></p> <p><i>Make a pyramid-shaped gift box of given dimensions.</i></p> <p>(b) Use properties of shape to justify explanations.</p> <p>Explaining and using properties associated with intersecting and parallel lines and triangles and knowing associated language.</p>	<p>(a) Use a computer database to draw conclusions.</p> <p>Inserting and interrogating data in a computer database; drawing conclusions.</p> <p><i>Draw conclusions from census data about the effect of an epidemic/industrial revolution/changes in transport.</i></p> <p>(b) Design and use an observation sheet to collect data.</p> <p>Designing and using an observation sheet to collect data; collating and analysing results.</p> <p>Collecting, ordering and grouping continuous data using equal class intervals and creating frequency tables.</p> <p><i>Conduct a survey of cars passing with one, two, three, ... occupants.</i></p> <p><i>Devise a simple habitat recorder for an ecological survey.</i></p>
LEVEL 5			<p>(a) Find a sequence in which the third or any subsequent number is the sum of the previous two numbers.</p> <p>Interpret instructions written for a programmable toy.</p>	<p>(a) Give reasons when identifying equal angles in a diagram.</p> <p><i>Find the centres, axes and planes of symmetry in a variety of plane and solid shapes.</i></p> <p>(b) Use networks to solve problems.</p> <p>Using networks to solve problems.</p> <p>Specifying location by means of coordinates in four quadrants.</p> <p><i>Find the shortest route for a person delivering the post.</i></p>

**BASIC
TIER**

ATTAINMENT TARGET 2 ATTAINMENT TARGET 3 ATTAINMENT TARGET 4 ATTAINMENT TARGET 5

NUMBER ALGEBRA SHAPE AND SPACE

- (c) **Refine estimations by 'trial and improvement' methods.**
Using 'trial and improvement' methods.
Approximating, using significant figures or decimal places.
Estimate the square root of 10 and refine to 3 decimal places.
- (d) **Use units in context.**
Using Imperial units still in daily use and knowing their rough metric equivalents.
Converting one metric unit to another.
Using negative numbers in context, including ordering, addition, subtraction and simple multiplication and division.
Using index notation to express powers of whole numbers.
Use in estimating, that 1 kg is about 2 lb, 8km is approximately 5 miles, 1 litre is about 1.75 pints.

Work out that 2.4 kg is equivalent to 2400g.
Calculate the increase in temperature from -4 °C (4 degrees of frost) to +10 °C.

LEVEL 5

- (b) **Express a simple function symbolically.**
Expressing simple functions symbolically.
Understanding and using simple formulae or equations expressed in symbolic form.
Understanding and using co-ordinates in all four quadrants.
Write the total cost, c pence, of n cakes as $c = 15 \times n$ (or $15n$) when the cost of the cake is 15p.
- (d) **Use the fact that the perimeter p of a rectangle is given by $p = 2(a + b)$ where a and b are the dimensions.**

Using index notation to express powers of whole numbers.

Use in estimating, that 1 kg is about 2 lb, 8km is approximately 5 miles, 1 litre is about 1.75 pints.

Work out that 2.4 kg is equivalent to 2400g.

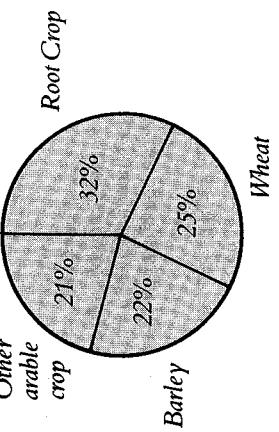
Calculate the increase in temperature from -4 °C (4 degrees of frost) to +10 °C.

ATTAINMENT TARGET 4 ATTAINMENT TARGET 5

HANDLING DATA

- (c) **Find areas of plane shapes or volumes of simple solids.**
Finding areas of plane figures (excluding cylinders), using appropriate formulae.
Finding volumes of regular solids (excluding cylinders), using appropriate formulae.
Finding the circumference of circles, practically, introducing the ratio π .
Know and use the formulae for finding the areas of squares, rectangles, triangles. Find the volumes of cubes, cuboids and triangular prisms.
- (d) **Find areas of plane shapes or volumes of simple solids.**
Constructing and interpreting conversion graphs.
Constructing and interpreting frequency diagrams and choosing class intervals for a continuous variable.
Use foreign exchange conversion graphs to change from, say dollars to marks.

Interpret this pie chart:



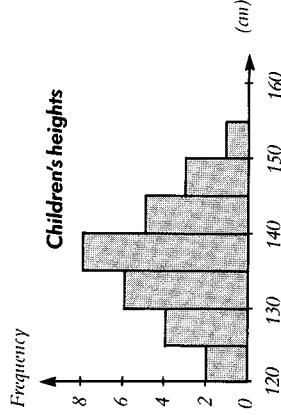
Farming Statistics

Interpret the graph of heights of pupils in the class and say whether this is the shape of the graph that they might expect. Explain why.

Class intervals	Frequency
$120 \leq h < 125$	2
$125 \leq h < 130$	4
$130 \leq h < 135$	6
$135 \leq h < 140$	8
$140 \leq h < 145$	5
$145 \leq h < 150$	3
$150 \leq h < 155$	1

h = height (centimetres)

ATTAINMENT TARGET 5
HANDLING DATA



(d) Use an appropriate method for estimating probabilities.

Understanding that different outcomes may result from repeating an experiment.

Recognising situations where estimates of probability can be based on equally likely outcomes and others where estimates must be based on statistical evidence.

Knowing that if each of n events is assumed to be equally likely, the probability of one occurring is $1/n$.
Understand that if the names of 5 people are put into a bag the probability of picking a particular name is $1/5$.

Decide that an estimate of the probability that the next vehicle passing the school would be carrying one passenger could be made by first doing a traffic survey.
Realise that the chance of winning a game of hockey is not necessarily $1/3$ (for WIN, LOSE, DRAW), and consider other ways of estimating probable outcomes.

Realise that equally likely assumptions are not appropriate for estimating probabilities in some situations, eg the probability that the driver of the next car passing the school will be a woman will depend on the proportion of women drivers to men drivers in the area.

BASIC TIER	ATTAINMENT TARGET 2 NUMBER	ATTAINMENT TARGET 3 ALGEBRA	ATTAINMENT TARGET 4 SHAPE AND SPACE	ATTAINMENT TARGET 5 HANDLING DATA
AND TIER	Pupils should be able to: Pupils should engage in activities which involve: <i>Pupils could:</i>	Pupils should be able to: Pupils should engage in activities which involve: <i>Pupils could:</i>	Pupils should be able to: Pupils should engage in activities which involve: <i>Pupils could:</i>	Pupils should be able to: Pupils should engage in activities which involve: <i>Pupils could:</i>
CENTRAL TIER	<p>(a) Calculate with fractions, decimals, percentages or ratio as appropriate.</p> <p>Ordering decimals and appreciating place values.</p> <p>Understanding and using equivalent fractions and equivalent ratios and relating these to decimals and percentages.</p> <p>Working out fractional and percentage changes.</p> <p>Converting fractions to decimals and percentages and finding one number as a percentage of another.</p> <p>Calculating, using ratios in a variety of situations.</p> <p><i>Adapt a recipe for 6 people to one for 8 people.</i></p> <p><i>Find out which is the better buy: trainers in one shop at 20% off or the same trainers in a shop at three-quarters of their original price.</i></p> <p><i>Explore relationships between fractions and decimals.</i></p> <p>(b) Enlarge a design in a given ratio.</p> <p>Use estimation to check calculations.</p>	<p>(a) Explore number patterns using computer facilities or otherwise.</p> <p>Using spreadsheets or other computer facilities to explore number patterns. Suggesting possible rules for generating sequences.</p> <p><i>Use the difference method to explore sequences such as: 2, 5, 10, 17, 26 ...</i></p> <p>(b) Solve simple equations.</p> <p>Solving linear equations; solving simple polynomial equations by 'trial and improvement' methods.</p> <p><i>Solve: $3x + 4 = 10 - x$</i></p> <p><i>Solve equations such as $x^2 = 5$ and $x^3 = 20$ by 'trial and improvement' using a calculator.</i></p>	<p>(a) Use 2-D representation of 3-D objects.</p> <p>Recognising and using common 2-D representation of 3-D objects.</p> <p><i>Use isometric paper to represent 3-D objects.</i></p> <p>(b) Transform shapes using a computer, or otherwise.</p> <p>Enlarging a shape by a whole number scale factor.</p> <p>Classifying and defining types of quadrilaterals.</p> <p>Knowing and using angle and symmetry properties of quadrilaterals and other polygons.</p> <p>Using computers to generate and transform 2-D shapes.</p>	<p>(a) Design and use a questionnaire to survey opinion.</p> <p>Specifying an issue for which data are needed; designing and using observation sheets to collect data; collating and analysing results.</p> <p>Designing and using a questionnaire to survey opinion (taking account of bias); collating and analysing results.</p> <p><i>Conduct a survey of taste in poetry, music, literature, art, television programmes, etc.</i></p> <p><i>Determine the best of three possible locations for a proposed new youth club.</i></p> <p><i>Conduct a survey of people's opinions about the causes of, and remedies for, world hunger.</i></p>

LEVEL 6

- (c) **Use and plot Cartesian coordinates to represent mappings.**
- Drawing and interpreting simple mappings in context, recognising their general features.
- Explore relationships between fractions and decimals.*
- (b) **Enlarge a design in a given ratio.**
- Use estimation to check calculations.**
- Using estimation and approximation to check that answers to multiplication and division problems involving whole numbers are of the right order.
- Estimate that $278 + 39$ is about 7.*

$$\begin{aligned}x \rightarrow x + 1 & (\text{or } y = x + 1) \\x \rightarrow 2 - x & (\text{or } y = 2 - x) \\x \rightarrow x^2 & (\text{or } y = x^2)\end{aligned}$$

Use bearings in real-life examples, such as describing the position of a ship or aircraft or the location of a buoy.

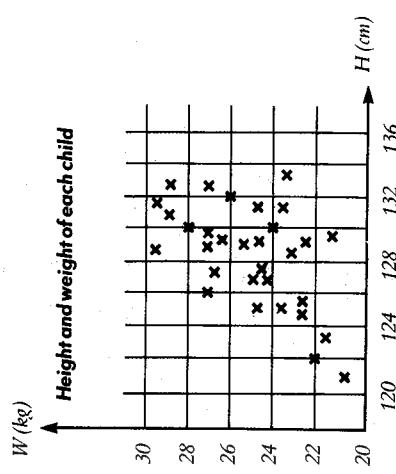
**BASIC
TIER**

**ATTAINMENT TARGET 4
SHAPE AND SPACE**

**ATTAINMENT TARGET 5
HANDLING DATA**

- (d) Demonstrate that they know and can use the formulae for finding the areas and circumferences of circles.
- Finding areas of circles using the formula.
Find the radius of a circle with the same area as a square of side 5cm.
- Calculate how many times the wheels of a bicycle rotate in a journey of 5km if the radius of each wheel is 34cm.

- (b) Understand and use the basic ideas of correlation.
- Creating scatter graphs for discrete and continuous variables and having a basic understanding of correlation.
- Constructing and interpreting information through two-way tables and network diagrams.
- Draw a scatter graph to show the correlation between life expectancy and accessibility to clean water, for a variety of richer and poorer countries.
- Comment on the relationship shown between height and weight.



LEVEL 6

**BASIC
TIER**
AND
**CENTRAL
TIER**

**ATTAINMENT TARGET 5
HANDLING DATA**

- (c) Identify all the outcomes of combining two independent events.

Identifying all the outcomes when dealing with two combined events which are independent using diagrammatic, tabular or other forms.

List all the outcomes when tossing 2 coins, HH, TT, TH, HT.

List all the outcomes when tossing two dice and show the total sums arising.

- (d) Know that the total probability of all the mutually exclusive outcomes of an event is 1.

Appreciating that the total sum of the probabilities of mutually exclusive events is 1 and that the probability of something happening is 1 minus the probability of it not happening.

Recognise that if the probability of a machine failing is 0.05 then the probability of it not failing is 0.95.
Determine what the probability is of drawing a green ball from a bag of 50 balls of 4 different colours (5 green, 15 blue, 20 yellow and 10 red); also the probability of not drawing a green ball.

LEVEL 6

CENTRAL NUMBER TIER	ATTAINMENT TARGET 2 ATTAINMENT TARGET 3 ALGEBRA	ATTAINMENT TARGET 4 SHAPE AND SPACE	ATTAINMENT TARGET 5 HANDLING DATA
AND TIER	<p>Pupils should be able to: Pupils should engage in activities which involve: <i>Pupils could:</i></p> <p>(a) Multiply and divide mentally single digit multiples of any power of 10.</p> <p>Multiplying and dividing mentally single-digit multiples of any power of 10, realising that with a number less than 1, multiplication has a decreasing effect and division has an increasing effect.</p> <p><i>Work out mentally</i> 80×0.2 and $600 + 0.2$.</p> <p>(b) Use a calculator efficiently when solving problems.</p> <p>Solving problems using multiplication and division with numbers of any size.</p> <p>Expressing positive integers as a product of primes.</p> <p>Using the memory and bracket facilities of a calculator to plan a calculation and evaluate expressions.</p> <p><i>Use a calculator to convert inches to centimetres given that there are 0.394 inches to the centimetre.</i></p> <p><i>Use a calculator to find which countries use most and least of the world's energy resources per capita.</i></p> <p><i>Evaluate formulae arising in other areas of the curriculum</i></p>	<p>Pupils should be able to: Pupils should engage in activities which involve: <i>Pupils could:</i></p> <p>(a) Use symbolic notation to express the rules of sequences.</p> <p>Using symbolic notation to express the rules of sequences.</p> <p>Exploring complex number patterns generated by a computer.</p> <p>Using the rules of indices for positive integer values.</p> <p>Understanding the meaning of reciprocals and exploring relationships.</p> <p><i>Express in symbols the rules for the following sequences:</i> $1, 3, 5 \dots [2n - 1]$ $1, 4, 9 \dots [n^2]$ $\chi_1, \chi_2, \chi_3 \dots [n(n + 1)]$</p> <p>(b) Solve equations or simple inequalities.</p> <p>Solving a range of polynomial equations by 'trial and improvement' methods.</p> <p>Using algebraic and graphical methods to solve simultaneous equations in two variables.</p> <p>Drawing and interpreting the graphs of linear functions.</p> <p>Generating various types of graphs on a computer or calculator and interpreting them.</p> <p><i>Constructing and interpreting flow diagrams with and without loops.</i></p>	<p>Pupils should be able to: Pupils should engage in activities which involve: <i>Pupils could:</i></p> <p>(a) Organise and analyse data.</p> <p>Specifying a simple hypothesis; designing and using an appropriate questionnaire or method to test it; collecting and analysing results to see whether a hypothesis is valid.</p> <p>Using and recording grouped data with class intervals suitably defined; producing a frequency table; calculating the mean using a calculator.</p> <p>Comparing the mean, median, mode and range of a frequency distribution where appropriate for given sets of data and interpreting the results.</p> <p><i>Find the locus of points equidistant from two fixed points.</i></p> <p><i>Find the locus of points whose sum of distances from two fixed points is constant.</i></p> <p><i>Predict relative movements of parts in a simple linked-cog or pulley system.</i></p> <p>(c) Use Pythagoras' theorem.</p> <p>Understanding and applying Pythagoras' theorem.</p> <p><i>Find the distance between two points expressed as co-ordinates.</i></p> <p>(d) Carry out calculations in plane and solid shapes.</p> <p>Using knowledge and skills in length, area and volume to carry out calculations in plane and solid shapes.</p> <p>Englaring a shape by a fractional scale factor.</p> <p><i>Calculate lengths, areas and volumes in shapes involving rectangles, triangles, parallelograms, circles, trapezia, cubes, cuboids, cylinders and other solids of constant cross-sectional area.</i></p>
FURTHER TIER	<p>(a) Use co-ordinates (x, y, z) to locate position in 3-D.</p> <p>Using co-ordinates to locate position in 3-D.</p> <p><i>Find some of the possible locations of the vertices of a cuboid of dimensions 4, 2, 1 if the position of one vertex is (3, 2, 0).</i></p> <p>(b) Determine the locus of an object which is moving subject to a rule.</p> <p>Determining the locus of an object moving subject to a rule.</p> <p><i>Find the locus of points equidistant from two fixed points.</i></p> <p><i>Find the locus of points whose sum of distances from two fixed points is constant.</i></p> <p><i>Predict relative movements of parts in a simple linked-cog or pulley system.</i></p> <p>(c) Use Pythagoras' theorem.</p> <p>Understanding and applying Pythagoras' theorem.</p> <p><i>Find the distance between two points expressed as co-ordinates.</i></p> <p>(d) Carry out calculations in plane and solid shapes.</p> <p>Using knowledge and skills in length, area and volume to carry out calculations in plane and solid shapes.</p>	<p>(a) Organise and analyse data.</p> <p>Specifying a simple hypothesis; designing and using an appropriate questionnaire or method to test it; collecting and analysing results to see whether a hypothesis is valid.</p> <p>Using and recording grouped data with class intervals suitably defined; producing a frequency table; calculating the mean using a calculator.</p> <p>Comparing the mean, median, mode and range of a frequency distribution where appropriate for given sets of data and interpreting the results.</p> <p>Drawing a frequency polygon as a line graph from a frequency distribution for grouped data; making comparisons between two frequency distributions.</p> <p>Constructing and interpreting flow diagrams with and without loops.</p> <p>Drawing a line of 'best fit' by inspection on a scatter diagram.</p>	

LEVEL 7

**CENTRAL
TIER**

**ATTAINMENT TARGET 2 ATTAINMENT TARGET 3
NUMBER ALGEBRA**

(c) Recognise that measurement is approximate and choose the degree of accuracy appropriate for a particular purpose.

Recognising that measurement is approximate and choosing the degree of accuracy appropriate for a particular purpose.

Recognising that a measurement expressed to a given unit is in possible error of half a unit.

Understanding and using compound measures, eg speed, density.

Know that a distance of 10 kilometres lies between 9.5 and 10.5 kilometres and a time of 9.57 seconds lies between 9.565 and 9.575 seconds.

Choose appropriate measuring instruments to measure volume, eg a burette rather than a measuring cylinder for certain purposes in science.

List the values of n where n is a whole number such that:

$$-9 < n \leq 20.$$

Solve: $x^2 + x = 5$

Solve: $\begin{aligned} 2x - y &= 9 \\ x + 3y &= 8 \end{aligned}$

Use travel graphs to solve distance/time problems.

Solve graphically:

$$2x + y = 40$$

$$3x - 2y = 4$$

Find the largest whole number less than the cube root of a positive whole number N :

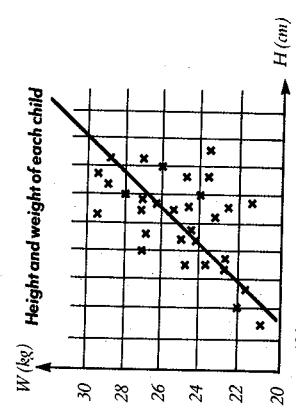
LEVEL 7

**ATTAINMENT TARGET 5
HANDLING DATA**

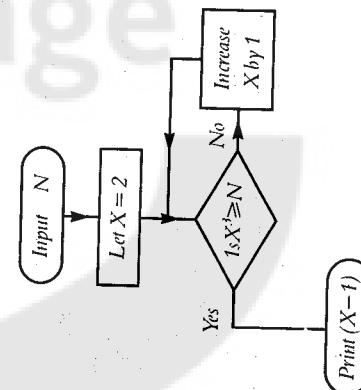
Compare the mean heights of sets of children of different ages and interpret. Organise and analyse examination marks: range 0-100, intervals of 10 marks:

Marks	Mid-point	Frequency
0-9	4.5	2
10-19	14.5	5
20-29	24.5	15
30-39	34.5	24

Draw a line of 'best fit':



Compare the proportion of foreign news in a number of issues of a range of daily newspapers or news bulletins.



CENTRAL
TIER
AND
FURTHER
TIER

ATTAINMENT TARGET 5
HANDLING DATA

- (b) Understand and use relative frequency as an estimate of probability.

Understanding and using frequency as an estimate of probability.

Apreciating, when assigning probabilities that relative frequency and equally likely considerations may not be appropriate and 'subjective' estimates of probability have to be made.

Recognise that when a 'fair' die is rolled many times each number will appear on approximately $\frac{1}{6}$ of the total number of throws and expect that the proportion will generally approximate to $\frac{1}{6}$ as the number of throws is increased. Use a computer simulation to test this hypothesis.

- (c) Given the probability of exclusive events calculate the probability of a combined event.

Understanding and applying the addition of probabilities for mutually exclusive events.

Check that events are mutually exclusive and then add their probabilities to calculate the probability of a combined event.

LEVEL 7

CENTRAL TIER

ATTAINMENT TARGET 2 ATTAINMENT TARGET 3 ATTAINMENT TARGET 4 ATTAINMENT TARGET 5

Pupils should be able to:
 Pupils should engage in activities which involve:
Pupils could:

(a) Calculate with numbers expressed in standard form.
 Expressing and using numbers in standard index form, with positive and negative integer powers of 10.
 Using index notations to represent powers and roots.
 $\text{Calculate } (3.2 \times 10^4) + (1.6 \times 10^{-3}).$
 Evaluate compound measures involving large numbers arising in branches of science, eg nutrient levels in ecosystems.

(b) Evaluate formulae including the use of fractions or negative numbers.
 Substituting negative numbers into formulae involving addition, subtraction, multiplication and division.
 Calculating with fractions.
 $\text{Work out } s = ut + \frac{1}{2}at^2 \text{ when } t = 0.5,$
 $a = -3, u = 8.$

(c) Solve numerical problems, checking that the results are of the right order of magnitude.
 Estimating and approximating to check that the results of calculations are of the right order.
 Recognise that $(0.25 \times 83.4) + 5.7$ is about 3 or 4.

$$3n + 4 < 17,$$

$$x^2 \leq 16,$$

$$x < 10, y > 6, y < 2x + 3$$

Pupils should be able to:
 Pupils should engage in activities which involve:
Pupils could:

(a) Manipulate algebraic formulae, equations or expressions.
 Manipulating algebraic expressions.
 Understanding and using a range of formulae and functions.
 Understanding the relationship between powers and roots.
 Understanding direct and inverse proportions.
 Interpreting and using m and c in $y = mx + c.$
 Find common factors such as:
 $a^2x + ax^2 = ax(a + x)$
 Transform formulae such as:
 $V = IR, v = u + at$
 Multiply out two brackets $(ax + b)(cx + d)$
 $\text{Use the formula } T = 2\pi/\text{l}(\text{g}) \text{ to calculate one variable, given the other.}$
 Work with direct proportion, inverse proportion and inverse square law.

(b) Solve inequalities.
 Solving a variety of linear and other inequalities.
 Using straight-line graphs to locate regions given by linear inequalities.
 Solve:
 $3n + 4 < 17,$
 $x^2 \leq 16,$
 $x < 10, y > 6, y < 2x + 3$
 $\text{and } r(\pi + 2) \text{ those which denote (say) volume.}$

Pupils should be able to:
 Pupils should engage in activities which involve:
Pupils could:

(a) Design and use a questionnaire or experiment to test a hypothesis.
 Designing and using a questionnaire with multiple responses or an experiment with several variables, collating and analysing results to test a hypothesis.
 Analyse findings from a survey in which respondents have listed a set of products in order of preference.
 Analyse the germination rate of seeds of the same species under different conditions.

(b) Construct and interpret a cumulative frequency curve.
 Constructing a cumulative frequency table.
 Constructing a cumulative frequency curve using the upper boundary of the class interval, finding the median, upper quartile, lower quartile and inter-quartile range, and interpreting the results.
 Construct and comment on a cumulative frequency curve of a set of examination results:

**CENTRAL
TIER**

AND

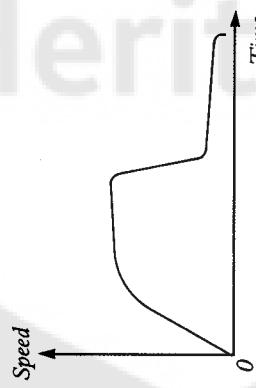
**FURTHER
TIER**

**ATTAINMENT TARGET 3
ALGEBRA**

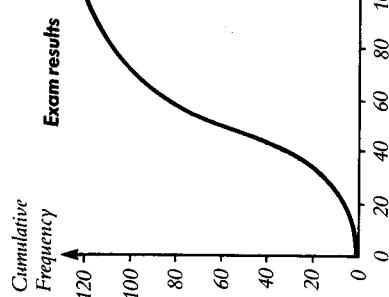
- (c) Interpret graphs which represent particular relationships.

Knowing the form of graphs of simple functions, eg quadratic, cubic, reciprocal. Interpreting graphs which describe real-life situations and contexts.

Suggest a sport which could produce a graph like this. Explain their choice.



**ATTAINMENT TARGET 5
HANDLING DATA**



- (c) Calculate the probability of a combined event given the probabilities of independent events.

Calculating the probability of a combined event given the probability of two independent events and illustrating combined probabilities of several events using tabulation or tree-diagrams.

Understanding that when dealing with two independent events, the probability of them both happening is less than the probability of either of them happening (unless the probability is 0 or 1).

Find the probability of a cyclist having to stop at one set of lights, using a tree-diagram or otherwise, if there are 2 sets of traffic lights on the way to school and the probabilities of getting straight through the lights without having to stop are 0.6 and 0.4, respectively.

Draw a tree-diagram, or use a tabulation to define all of the possible outcomes of 3 events, where each event has 2 outcomes, eg tossing a coin 3 times.

LEVEL 8

FURTHER TIER

ATTAINMENT TARGET 2 ATTAINMENT TARGET 3 ATTAINMENT TARGET 4 ATTAINMENT TARGET 5

NUMBER ALGEBRA SHAPE AND SPACE HANDLING DATA

Pupils should be able to:

Pupils should engage in activities which involve:
Pupils could:

(a) **Distinguish between rational and irrational numbers.**
Distinguishing between rational and irrational numbers.
Know that $\sqrt{2}$ and π are irrational.
Know the significance of recurring and non-recurring decimals in this context.

(b) **Understand the significance of approximations.**
Understanding upper and lower bounds of numbers expressed to a given degree of accuracy.
Know the difference between 4.6 and 4.60 as decimals.
Realise that a number written as 9.7 correct to one decimal place can actually lie anywhere between 9.65 and 9.75 and be 9.65 or 9.75 depending on conventions.

LEVEL 9

Pupils should be able to:
Pupils should engage in activities which involve:
Pupils could:

(a) **Express general laws in symbolic form.**
Expressing general laws in symbolic form.
Using rules of indices for negative and fractional values.
Use $x^0 = 1$, $y^{-3} = 1/y^3$, $x^2/x^3 = 1/x = x^{-1}$ where neither x nor y is zero.
Find an approximate formula connecting S and A from a table of shutter speed (S) for the given apertures (A) on a camera.

(b) **Solve equations using graphical methods.**
Solving equations using graphical methods.
Constructing growth and decay rates and displaying them graphically.
Use the graph of $y = x^2 + 5x$ and $y = x^3$ to solve $x^3 = x^2 + 5x$.

(c) **Use the gradients of graphs found by constructing tangents.**

Constructing tangents to graphs to determine the gradient.
Find velocity in distance/time graphs and acceleration in velocity/time graphs.

Use tangents to investigate the gradient of $y = x^2$.

Pupils should be able to:
Pupils should engage in activities which involve:
Pupils could:

(a) **Carry out more complex calculations in plane or solid shapes.**
Calculating distances and angles in solids using plane sections and trigonometric ratios.
Understanding the conditions for congruent triangles.
Understanding and using the relationship between surface areas of similar figures and volume of similar 3-D solids.
Calculating lengths of circular arcs and areas of shapes whose perimeters include circular arcs; calculating surface area of cylinders and volumes of cones and spheres.

Find the angle that the edge of a square-based pyramid makes with its base.
Prove that two triangles are congruent by reference to corresponding properties.
Make an accurate set of drawings for making a piece of jewellery incorporating several different polyhedra.

Find a histogram to illustrate the distribution of ages in a town. Discuss critically with reference to other local data and compare to national figures.

Pupils should be able to:
Pupils should engage in activities which involve:
Pupils could:

(a) **Use diagrams, graphs or computer packages to analyse a set of complex data.**
Constructing and interpreting a histogram, with understanding of the connection between area and frequency.
Presenting a set of complex data in a simplified form using a variety of diagrams and graphs and computer statistical packages.
Use genuine data to examine the relative importance of adult literacy and good nutrition to reduction of infant mortality in a range of countries.

Design and carry out a statistical investigation to determine whether estimation skills are improved by practice.
Make a table from a government report or an opinion poll and prepare it for publication in the popular press.
Use a histogram to illustrate the distribution of ages in a town. Discuss critically with reference to other local data and compare to national figures.

ATTAINMENT TARGET 4 ATTAINMENT TARGET 5

SHAPE AND SPACE HANDLING DATA

Explain how many interlocking cubes will be needed to enlarge a model made with seven cubes by a given scale factor.

Calculate the area and perimeter of a sector of a circle (radius 12cm) when the angle at the centre is 35 degrees.

(b) **Use vector methods to solve problems.**
Understanding and using the laws of addition and subtraction of vectors.
Find the resultant force when an object is pulled in two directions.

Calculate the wind speed given the actual velocity of an aeroplane and its velocity in still air.

(c) **Use sine, cosine or tangent with angles of any size.**
Finding sine, cosine and tangent of angles of any size.

Sketching the graphs of sine, cosine and tangent functions for all angles.
Generating trigonometric functions using a calculator or computer and interpreting them.

Draw the graphs of $y = 10 \sin x^\circ$, $y = 6$ and $10y + x = 50$. Use these to solve the equations

$$5 \sin x^\circ = 3 \text{ and } 100 \sin x^\circ = 50 - x.$$

LEVEL 9

(b) **Use sampling to investigate a 'population'.**

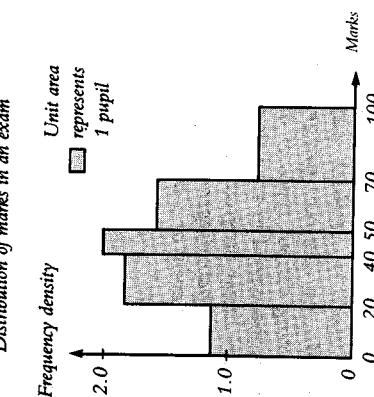
Using sampling to investigate a 'population' and recognising the reliability of different methods in relation to different sizes of population.

Take a simple or stratified sample of pupils in the school to investigate television viewing behaviour.

(c) **Use conditional probabilities.**

Producing a tree-diagram to illustrate the combined probability of several events which are not independent.

Calculate the probability that an operation will fail twice, given that it has a 60% success rate the first time it is attempted and if it is unsuccessful it can be repeated, but with a success rate of only 30%.



Unit area

■ represents
1 pupil

FURTHER TIER

ATTAINMENT TARGET 2 ATTAINMENT TARGET 3 ATTAINMENT TARGET 4 ATTAINMENT TARGET 5

NUMBER ALGEBRA SHAPE AND SPACE HANDLING DATA

Pupils should be able to:
 Pupils should engage in activities which involve:
Pupils could:

(a) **Determine the possible effects of errors on calculations.**
 Calculating the upper and lower bounds in the addition, subtraction, multiplication and division of numbers expressed to a given degree of accuracy.
 Determining the possible effects of error on calculations involving measurements.
 Given the length and breadth of a rectangle, calculate the range within which the area lies.
 Recognise that calculating the 'weight' of 100 identical objects, given the 'weight' of only one of them, magnifies the possible error.

LEVEL 10

(a) **Use a calculator or computer to investigate sequences.**
 Using a calculator or computer to investigate whether a sequence given iteratively converges or diverges.
Investigate sequences formed by calculating the mean of a previous set of numbers in the sequence such as 1, 2, 3, 2, 2½... (where each term is the mean of the three previous numbers).
 (b) **Manipulate algebraic expressions where necessary when solving problems.**
 Manipulating a range of algebraic expressions in a variety of contexts.
*Factorise algebraic expressions which arise in problems.
 Solve quadratic equations by using factors, the common formula, completing the square or iteration as appropriate.*

Simplify:

$$1/(x+2) + 1/(x-3)$$

Show that:

$$x^2 - 6x + 10 = (x - 3)^2 + 1 \geq 1$$

Pupils should be able to:
 Pupils should engage in activities which involve:
Pupils could:

(a) **Solve problems in 2-D or 3-D.**
 Knowing and using angle and tangent properties of circles.
 Using sine and cosine rules to solve problems including simple cases in 3-D.
 Understanding how transformations are related by combinations and inverses.
 Using matrices to define transformations in 2-D.
Using computer software or otherwise, find one transformation which is equivalent to successive applications of two given transformations.
Calculate the distance between two ships if they move at constant speeds in the direction 060° and 100° respectively, the first ship moving at 20km/h and the second at 24km/h.

(a) **Describe the dispersion of a set of data.**
 Describing the dispersion of a set of data; calculating standard deviation of a set of data.
Considering different shapes of histograms representing distributions, with special reference to mean and dispersion, including the normal distribution.
Compare and contrast histograms of data which has been collected either directly or from published sources.

(b) **Interpret diagrams such as those used in critical path analysis or linear programming.**
Interpreting various types of diagrams such as those used in analysis of critical path and linear programming.
Use critical path diagrams in preparing for a social event or a business venture.

FURTHER
TIER

ATTAINMENT TARGET 3
ALGEBRA

(c) Find the approximate area between a curve and the horizontal axis between two limits, and interpret the result.

Finding the approximate area between a curve and the horizontal axis between two limits, and interpreting the result.
Understand and use the fact that the area under a velocity/time graph represents the distance travelled.

(d) Sketch and compare the graphs of functions.

Sketching the graph of functions derived from other functions, eg $y = f(x - a)$,
 $y = f(kx)$, $y = f(x) + a$ from the graph of $f(x)$ for different values of a and k .
Interpreting and using co-efficients in quadratics.

Sketch the graph of $y = 1/x^2$, given the graph of $y = x^2$, checking the result using a computer or calculator.

Sketch $y = f(x - 3)$

$$y = f(4x)$$

$$y = f(x) - 8$$

given the graph of $y = f(x)$

where $f(x) = x - x^2$.

Sketch and compare $1/x$, $1/x^2$, $1/x^3$, $1/x^4$ checking results using computer or calculator.

ATTAINMENT TARGET 5
HANDLING DATA

(c) Calculate the probability of any two events happening.
Understanding the probability for any two events happening.

*Find the probability of:
obtaining balls of same colour when drawing two from a bag containing 3 red, 4 blue, 7 green balls;*

getting both questions hoped for in a test knowing the probability of each separately;

getting either a '5' or a 'head' (or both) when tossing a dice and a coin simultaneously.

LEVEL 10

ATTAINMENT TARGET 1

USING AND APPLYING MATHEMATICS

Pupils should be able to:

Pupils should engage in activities which involve:

Pupils could:

BASIC TIER

- (a) **Find ways of overcoming difficulties when solving problems.**
Selecting the materials and the mathematics to use for a task using alternative approaches to overcome difficulties.
Work out that ten 9's are 90 and add three 9's to this when they need to multiply 13 by 9 to find number of tiles in the classroom.
When measuring orange juice, and the jug looks like overflowing, think of filling the jug, pouring into a bowl, measuring the rest and adding the two results together.
Design and make a weather vane which involves symmetry; test and modify.

LEVEL 3

- (b) **Use or interpret appropriate mathematical terms and mathematical aspects of everyday language in a precise way.**
Explaining work and recording findings systematically.
Explain methods of calculation using correct terminology.
Describe their model train using names for 3-D and 2-D shapes.
Read the instructions for a dice game involving words like 'multiply', 'add' and 'total', and explain to others in a group what to do.
Describe the weather vane and its function using appropriate terms.
- (c) **Present results in a clear and organised way.**
Keep a record of the weather over a period of time; display the results in an appropriate chart.
Record their results clearly without omissions when working out which numbers are in both the 3 and 4 times tables.
- (d) **Investigate general statements by trying out some examples.**
Investigating and testing predictions and general statements.
Checking results, considering whether they are sensible.
Investigate the possible total amount of money they might have when told someone has exactly three coins.
Explore what happens when you add pairs of odd and even numbers.

LEVEL 4

- (a) **Identify and obtain information necessary to solve problems.**
Selecting the materials and the mathematics to use for a task when the information leaves opportunities for choice; planning work methodically.
Realise the need to measure the length and width of a car in order to be able to mark out the playground as a car park for a school show.
When trying to draw repeating patterns of different sizes using LOGO, realise the need for a procedure to incorporate a variable, and request and interpret instructions for doing this.
When trying to fit a report onto a space in the school newsletter, realise the need for a smaller font size, and request and interpret the instructions for changing the font size.
- (b) **Interpret situations mathematically, using appropriate symbols or diagrams.**
Recording finding and presenting them in oral, written or visual form.
Use co-ordinates to record the classroom layout.
Decide to use own symbols like FB for a telephone line connecting France with Britain when investigating the number of direct telephone lines needed to link up different numbers of countries.
Translate the problem of finding the number of 28p packets of crisps that can be bought for £5 into ... $500 \div 28$
... in order to use a calculator;
record the result as ...
 $500 \div 28 = 17.857142$
... and thus decide that the result is 17.
- (c) **Give some justification for their solutions to problems.**
Using examples to test solutions, statements or definitions.
Test the validity of statements such as: 'Rectangles with the same area have the same perimeter'; 'it is harder to get a 6 on a die than a 1.'
Explain the reasons for some of the features in their design for a ground floor flat for a disabled person.
- (d) **Make generalisations.**
Making generalisations or simple hypotheses.
Observe from data they have collected that woodlice prefer dark, damp conditions because more of them are found under stones, damp rubbish etc.

ATTAINMENT TARGET 1

USING AND APPLYING MATHEMATICS

Pupils should be able to:

Pupils should engage in activities which involve:

Pupils could:

BASIC TIER AND CENTRAL TIER

- (a) Carry through a task by breaking it down into smaller more manageable tasks.
Selecting the materials and the mathematics to use for a task; checking there is sufficient information; working methodically and reviewing progress.
Breaking tasks into smaller more manageable tasks.
Look for different sets of triangles in turn when investigating the triangles that can be made on a 3×3 pinboard or grid, eg those with two sides along the grid-lines, those with just one side along the grid-lines and those with no side along the grid-lines.
Organise a group investigating the 'best buy' in snack bars so that one pair look at price, one at taste, and one at nutritional value.
Investigate strategies of a game such as Ayo (Nigeria); Roundhead (Maori) or the Tiger game (Indian).
Design a board game.

LEVEL 5

- (b) Interpret information presented in a variety of mathematical forms.
Interpreting mathematical information presented in oral, written or visual form.
Discuss the findings of an article in a magazine or newspaper which incorporates different forms of presentation, eg text, tables, graphs.
Give an account to the rest of the class of their project investigating the cross-sections of boxes, using models, posters or transparencies to present their results.
- (c) Make a generalisation and test it.
Generalising from a number of particular examples and carrying out simple tests.
Having investigated the difference between six 2-digit numbers and their reverses (eg 82 and 28), make the conjecture that the difference is always in the 9 times table, and decide to check three other numbers to test it.
Having made the hypothesis that people who live further away from the supermarket spend more than those who live near, decide what has to be done to test this, and collect and analyse the necessary data.
Make and test statements such as 'Most cars passing the school are over 3 years old'; 'Buying products in larger quantities gives better value for money'.

LEVEL 6

- (a) Pose their own questions or design a task in a given context.
Designing a task and selecting the mathematics and resources; checking information and obtaining any that is missing; using 'trial and improvement' methods.
Design and make a devise to measure accurately a given period of time (eg 2 minutes).
List the problems they will need to solve, and go on to tackle one or more of them when faced with the problem of running a profitable hot-dog stall at the school fair.
After investigating the symmetry properties of different types of quadrilateral, decide to go on to look at triangles, pentagons and so on.
- (b) Examine critically the mathematical presentation of information.
Examining and presenting findings using oral, written or visual forms.
Improve their own write-up of a study of the feasibility of setting up a lunchtime computer games club in order to present the proposal to the school council.
Read a report on magic squares made by another group, and suggest improvements to sections which were not clear, or did not follow logically.
Examine the results of an extended investigation presented orally or for display by another group.
Examine critically the presentation of information in the media.
- (c) Make a generalisation giving some degree of justification.
Making and testing generalisations and simple hypotheses; defining and reasoning in simple contexts with some precision.
Make a hypothesis about armspan in relation to height and justify it by reference to a scatter diagram.
Give the path traced out by a point moving according to a given rule and illustrate by devising instructions for a computer.

ATTAINMENT TARGET 1

USING AND APPLYING MATHEMATICS

Pupils should be able to:

Pupils should engage in activities which involve:

Pupils could:

CENTRAL TIER AND FURTHER TIER

LEVEL 7

- (a) Follow new lines of enquiry when investigating within mathematics itself or when using mathematics to solve a real-life problem.
Following new lines of investigation using alternative methods to overcome difficulties; devising a mathematical task; working methodically within an agreed structure; using judgement in the use of given information, using 'trial and improvement' methods and reviewing progress.
While drawing the nets of regular solids to make different dice, decide to investigate how many regular solids there are.
As part of a project examining the best way/place to invest savings, decide to look at the effect of using different intervals for calculating interest payments.
- (b) Examine and comment constructively on generalisations or solutions.
Following a chain of mathematical reasoning; spotting inconsistencies.
Having found that, for a given volume, a cube is the shape with the least surface area, explain that only shapes with rectangular sides were considered, and that a sphere would probably have an even smaller surface area as it is even more 'squashed together'.
Explore the validity of statements such as: ' $\sqrt[3]{3}$ has a cycle of 6 repeating digits'. 'All quadrilaterals tessellate.' ' $\sqrt[n+1]{n}$ is always smaller than $\sqrt[n]{n} + 1$ '.
Design and use a questionnaire to test the hypothesis that pupils/parents would prefer the school day to start at 08.00 hours and finish at 14.00 hours without a lunch break.

LEVEL 8

- (a) Give logical accounts of work with reasons for choices made.
Devising and extending a mathematical task, making a detailed plan of the work; working methodically; checking information; considering whether results are of right order.
Decide where to put a telephone box in the locality, giving reasons for their decisions.
Consider the effects in various countries of the melting of polar ice-caps and consequent rise in sea levels. Compare areas of land which would be covered by a rise of 50m or 100m, eg Britain and Bangladesh.
Explain why they have chosen to limit their investigations of packaging shapes to those that are possible without having to use glue.
- (b) Understand the role of counter-examples in disproving generalisations or hypotheses.
Making statements of conjecture using 'if ... then ...'; defining, reasoning, proving and disproving, using counter-examples.
Construct an extended chain or argument using 'if ... then ...' appropriately.
Having made the conjecture that the median is always smaller than the mean, try successfully to find an example in which this is not true.
Having tried out several examples in order to investigate which is larger, $2n$ or $n-2$, and found $2n$ to be always greater, search for a counter-example to demonstrate that this is not always true.

ATTAINMENT TARGET 1

USING AND APPLYING MATHEMATICS

Pupils should be able to:

Pupils should engage in activities which involve:

Pupils could:

FURTHER TIER

LEVEL 9

- (a) Co-ordinate a number of features or variables in solving problems.
Designing, planning and carrying through a mathematical task to a successful conclusion.
Stating whether a conjecture is true, false or not proven; defining and reasoning; proving and disproving; using symbolisation; recognising and using necessary and sufficient conditions.
Given the task of conducting a survey on what local people feel about a new planning proposal, using a sample of only 100 people, decide on the best way of selecting a representative sample. Examine the relationship between the number of grid-points on the perimeter and the number of interior grid-points for different shapes.
- (b) Justify their solutions to problems involving a number of features or variables.
Justify their solutions to the problem of what size, and how many, chairs a cafe owner should order to equip a new cafe of specified shape, by referring to: the costs, the number of people likely to come in groups, the profit, and the undesirability of customers feeling cramped, or having to get up to let others past.
When reporting on the best way to stock and transport pipes, explain solutions in terms of costing, volume, surface area and stability.

LEVEL 10

- (a) Explore independently a new area of mathematics.
Designing, planning and carrying through a mathematical task to a successful conclusion; presenting alternative solutions and justifying the selected route.
Exploring, developing and using constructively an area of mathematics that is new to them.
Use books from the library in order to complete a project on matrices and transformations to present to the class.
Write a booklet for other pupils about the mathematics involved in bell-ringing, in particular how group theory can be applied to change-ringing.
Explore conditional probability.
Explore relationships between trigonometric functions.
- (b) Handle abstract concepts of proof and definition.
Giving definitions which are sufficient or minimal.
Using symbolisation with confidence; constructing a proof, including proof by contradiction.
Follow from a book Euclid's proof by contradiction that $\sqrt{2}$ is irrational, write out a similar proof for $\sqrt{3}$ and find why it breaks down for $\sqrt{4}$.
Prove that the first differences in the sequence of square numbers generate the sequence of odd numbers.
Find their own proof that the angle in a semi-circle is a right angle, and its converse, stating what prior results have been assumed.