O Level

Mathematics

Session: **1984 June**
Type: **Question paper**
Code: **4004**

MATHEMATICS

4004/1 HOME

ORDINARY LEVEL

SYLLABUS D

PAPER 1

(Two and a half hours)

All *questions may be attempted.*

Answers are to be written on the question paper in the spaces provided, and the question paper is to be handed in at the end of the examination.

If working is needed for any question, it must be shown in the space below that question.

NEITHER MATHEMATICAL TABLES NOR SLIDE RULES NOR CALCULATORS MAY BE BROUGHT INTO THE EXAMINATION ROOM

Questions 1 to 23 carry 3 marks each;

Questions 24 to 28 carry 5 marks each;

Question 29 carries 6 marks.

NEITHER MATHEMATICAL TABLES NOR SLIDE RULES NOR CALCULATORS MAY BE USED IN THIS PAPER.

1   Given that $x = 4$ and $y = -3$, evaluate

(i) $2x + 5y$,

(ii) $7 - y^2$,

(iii) $y(x - y)$.

Answer (i) ....

(ii) ....

(iii) ....

2   Calculate the exact value of

(i) $3 \cdot 1 \times 0 \cdot 07$,

(ii) $73 \cdot 2 \div 0 \cdot 4$.

Answer (i) ....

(ii) ....

3   Given that $x \leqslant 12\frac{1}{4}$, state the largest possible value of x if

    (i) x is an integer,

    (ii) x is a prime number,

    (iii) x is a rational number.

Answer (i) ....

(ii) ....

(iii) ....

4   (i) Factorise completely $18a^2b - 30ab^2$.

    (ii) Factorise $1 - p - 12p^2$.

Answer (i) ....

(ii) ....

5   Giving each answer as a fraction in its lowest terms, find the exact value of

    (i) $\frac{2}{3} \times \frac{5}{8}$,

    (ii) $(3\frac{1}{2} - 1\frac{2}{3}) \div 3\frac{1}{3}$.

Answer (i) ....

(ii) ....

6   (a) Express $0 \cdot 003186$ correct to 3 significant figures.

    (b) Express $52\,300$ in standard form.

    (c) Express $\frac{33}{80}$ as a decimal.

Answer (a) ....

(b) ....

(c) ....

7   Evaluate the matrix product

    (i) $\begin{pmatrix} 2 & 0 & 3 \\ 0 & 4 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$,

    (ii) $\begin{pmatrix} 2 & 5 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 1 & -5 \end{pmatrix}$.

Answer (i) ....

(ii) ....

8   (a) Calculate the exact value of

    (i) $(763 - 387) \div 4$,

    (ii) $6 - 8 \cdot 7$.

    (b) Given that $2^{16} \div 2 = 2^k$, write down the value of k.

Answer (a) (i) ....

(ii) ....

(b) $k =$ ....

9   (a) Express without brackets in its simplest form $(3p + 2)(5p - 4)$.

    (b) Solve the equation $2x + 3 = 4(x + 1)$.

Answer (a) ....

(b) $x =$ ....

10   For the distribution $5, 8, 12, 10, 5, 3, 7, 5, 20, 10,$ find

    (i) the mode,

    (ii) the mean,

    (iii) the median.

Answer (i) Mode ....

(ii) Mean ....

(iii) Median ....

11 (a) A lady buys an article marked at £6.40 but, in addition, has to pay 15% tax. Calculate the total amount that she has to pay.

(b) When a shopkeeper sells a camera for £84, he makes a profit of 12%. Calculate the cost price of the camera.

Answer (a) ....

(b) ....

12 Evaluate

(i) $36^{\frac{1}{2}}$, (ii) $81^{\frac{3}{4}}$, (iii) $(\frac{5}{3})^{-2}$.

Answer (i) ....

(ii) ....

(iii) ....

13 Solve the simultaneous equations

$$3x + 2y = -4,$$
$$x - 3y = 17.$$

Answer $x = $ ....

$y = $ ....

14

In the diagram, O is the centre of the circle and CT is the tangent at C. The chord BC is parallel to OT and OC meets BA at X.

Given that $A\widehat{O}C = 44°$, calculate

(i) $A\widehat{B}C$.

(ii) $O\widehat{T}C$.

(iii) $A\widehat{X}C$.
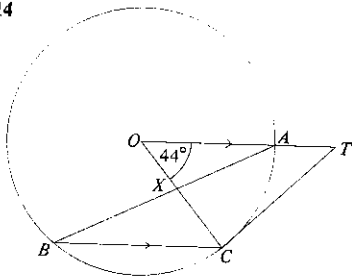
Answer (i) $A\widehat{B}C = $ ....

(ii) $O\widehat{T}C = $ ....

(iii) $A\widehat{X}C = $ ....

15 (a) Evaluate cos 115°, using as much of the information below as is necessary.

(b) In triangle PQR, $\widehat{Q} = 90°$, $\widehat{P} = 65°$ and $PQ = 4$ cm. Calculate QR, using as much of the information below as is necessary.

	25°	65°
sin	0·4226	0·9063
cos	0·9063	0·4226
tan	0·4663	2·145

Answer (a) cos 115° = ...

(b) $QR = $ .... cm

16 (a) Given that $\dfrac{2p}{x} = \dfrac{a}{b}$, find an expression for x in terms of a, b and p.

(b) Taking π to be $\frac{22}{7}$, calculate the radius of the base of a cylinder, given that its volume is 77 cm³ and its height is 8 cm.

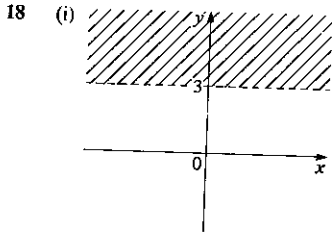*Answer* (a) $x = $ ....

(b) .... cm

17 P, Q, R and S are the four corners of a rectangular plot marked out on level ground. Given that the bearing of Q from P is 020° and the bearing of R from P is 090°, calculate the bearing of
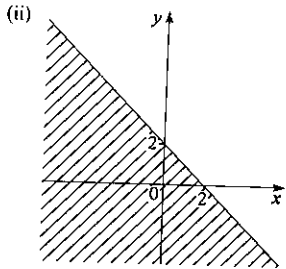
(i) P from Q,

(ii) R from Q,

(iii) S from Q.

Answer (i) ....

(ii) ....

(iii) ....

18 (i)

State the inequality which defines the **unshaded** area.

Answer (i) ....

(ii)

State the inequality which defines the **unshaded** area.

Answer (ii) ....

19 (*a*) Calculate the size of an exterior angle of a regular polygon with 20 sides.

(*b*)
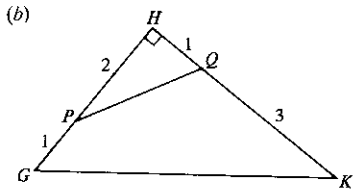
In the diagram, $GP = 1$ cm, $PH = 2$ cm, $HQ = 1$ cm, $QK = 3$ cm and $\widehat{H} = 90°$.

Calculate the area of the quadrilateral $GPQK$.

Answer (*a*) ....

(*b*) .... cm²

20 On the axes in the answer space, sketch, and label clearly, the graphs of

(i) $y = x - 1$,

(ii) $y = 1 - x^2$.

Answer
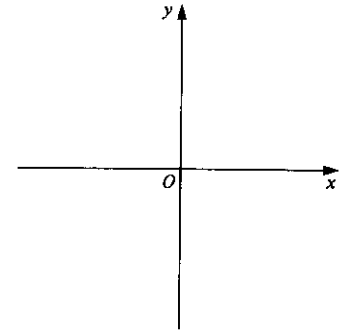
21 Express as a single fraction in its simplest form

$$\frac{3}{2x - 1} - \frac{4}{5x + 2}.$$

Answer ....

22 The sides of a triangle are of length 7 cm, 8 cm and 9 cm.

Calculate, as a fraction in its simplest form, the cosine of the angle opposite the 7 cm side.

Answer ....

23 A class of 30 children entered a competition in which the highest possible score was 6.
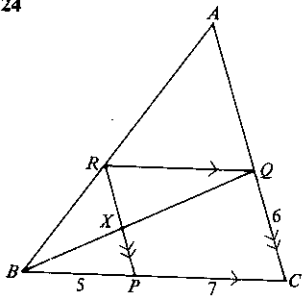
Their scores are given in the table.

Score	0	1	2	3	4	>4
No. of children	1	2	4	6	9	8

Draw clearly, on the axes in the answer space, a histogram to represent this data.

Answer
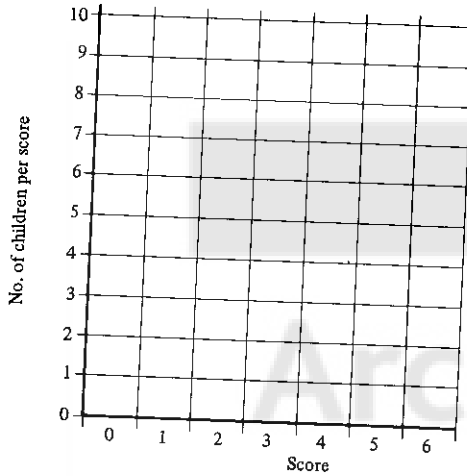
24

The points P, Q and R on the sides of triangle ABC are such that RQ is parallel to BC, RP is parallel to AC and BQ meets RP at X.

(i) Name two triangles, each of which is similar to triangle BQC.

(ii) Given that $BP = 5$ cm, $PC = 7$ cm and $QC = 6$ cm, use similar triangles to calculate

(a) XP,

(b) AQ.

Answer (i) .... and ....

(ii) (a) $XP =$ .... cm

(b) $AQ =$ .... cm

25 An aircraft flies from a point $P(75°N, 20°W)$ directly over the North Pole to a point Q.

(i) Given that the distance it flies from P to the North Pole is x nautical miles, calculate x.

(ii) Given that the further distance it flies from the North Pole to Q is $2x$ nautical miles, find (a) the latitude of Q, (b) the longitude of Q.

(iii) Another aircraft flies due east from P to a point R $(75°N, 30°E)$. Calculate, in nautical miles, the distance it flies, using as much of the information below as is necessary.

[$\sin 75° = 0.9659$, $\cos 75° = 0.2588$, $\tan 75° = 3.732$.]

Answer (i) $x =$ ....

(ii) (a) Latitude = ...

(b) Longitude = ...

(iii) .... nm

26 Two six-sided dice, one coloured black and one red, are thrown. Giving each answer as a fraction, calculate the probability that

(i) the score on the red die is 3,

(ii) each die shows a score of 5,

(iii) the score on the black die is either 2 or 4,

(iv) either the score on the black die is 1, or the score on the red die is 6, but not both.

Answer (i) ....

(ii) ....

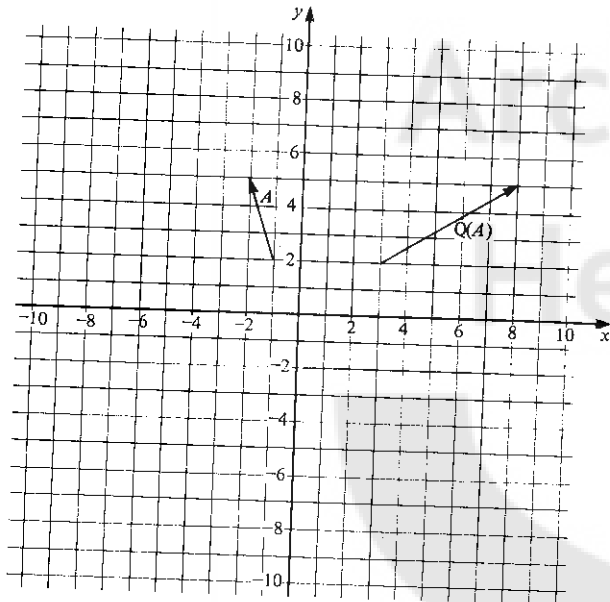(iii) ....

(iv) ....

27 P and Q are two transformations, P being represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

The diagram in the answer space shows an arrow A and its image under the transformation Q.

(i) Draw, and label clearly,

    (a) the arrow P(A),

    (b) the arrow PQ (A).

(ii) Find the 2×2 matrix which represents the transformation Q.

    Answer (i)

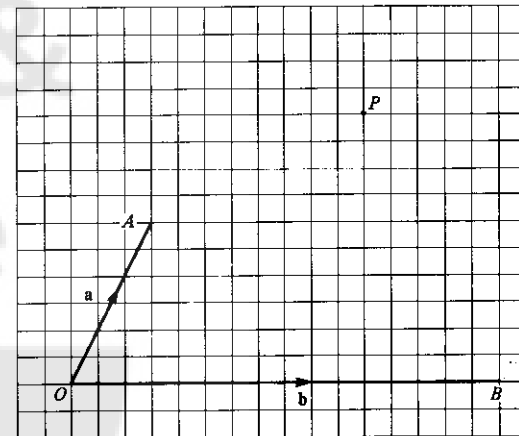   *(ii)* matrix is $\begin{pmatrix} & \\ & \end{pmatrix}$

28 The diagram in the answer space shows the origin O and three points A, B and P. The position vectors of A and B with respect to O are **a** and **b**.

Given that another point Q is such that

$$\overrightarrow{OQ} = h\mathbf{a} \quad \text{and} \quad \overrightarrow{QP} = k\mathbf{b},$$

(i) mark on the diagram, and label clearly, the point Q,

(ii) determine the value of

    (a) h,

    (b) k.

    Answer (i)

(ii) (a) h = ...

    (b) k = ...

29 Two towns, A and B, are 26 km apart. A cyclist leaves A at 13 00 and rides towards B at a steady speed of 16 km/h for 1 hour 15 minutes. He then rests until 15 00 and finally continues at a steady speed of v km/h, arriving at B at 15 30.

(i) Calculate v.

(ii) Calculate, in km/h, the average speed for the whole journey.

(iii) (a) Draw, on the axes in the answer space, the distance-time graph which represents the journey.

    (b) Given also that a lorry leaves B at 14 30 and travels to A at a steady speed of 20 km/h, draw, on the same axes, the graph representing this journey.

Answer (i) ....

          (ii) .... km/h

          (iii)
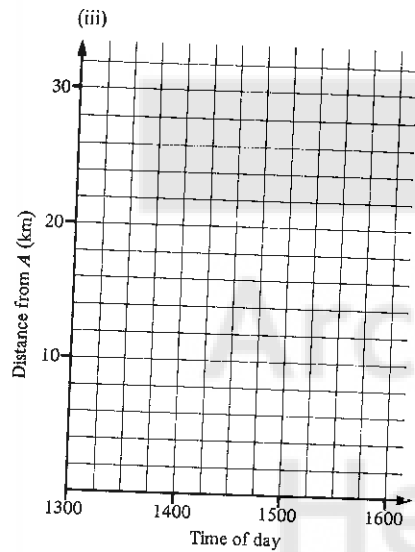
MATHEMATICS   4004/2 HOME

ORDINARY LEVEL

SYLLABUS D

PAPER 2

(Two and a half hours)

Answer **all** *the questions in Section A and any* **five** *questions from Section B.*

The intended marks for questions or parts of questions are given in brackets [ ].

All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.

If the degree of accuracy is not specified in the question and if the answer is not exact, three figure accuracy is required.

Mathematical tables or electronic calculators may be used to evaluate **explicit** *numerical expressions.*

Mathematical tables, graph paper and plain paper are provided.

Section A [40 *marks*]

Answer **all** *the questions in this section.*

1 (a)   Find the simple interest obtained when £125 is invested at 8% for a period of 9 months. [3]

   (b)   A sum of money is divided in the ratio 2:3:7. Given that the largest share is £112, calculate the smallest share. [3]

   (c)   The area of one face of a cube is 36 cm².

       Find (i) the volume of the cube,

            (ii) the total length of all its edges. [2]

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2 (a)   In a comprehensive school all 200 children in the first year study either Physics, or Chemistry, or both Physics and Chemistry.

       Given that 80% study Physics and 30% study Chemistry, find the number of children who study

       (i) both Physics and Chemistry,

       (ii) Physics only. [4]

   (b)   (i) Given that the determinant of the matrix $\begin{pmatrix} 4 & -5 \\ 1 & 2 \end{pmatrix}$ is equal to the determinant of the matrix $\begin{pmatrix} 5 & x \\ 3 & 8 \end{pmatrix}$, find the value of x.

         (ii) Find the inverse of the matrix $\begin{pmatrix} 4 & -5 \\ 1 & 2 \end{pmatrix}$. [4]


3
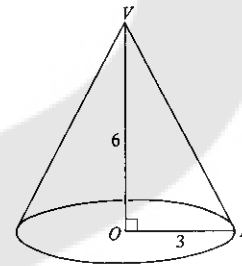
The diagram represents a circular cone of vertical height 6 cm standing on a horizontal base of radius 3 cm.

V is the vertex of the cone, O is the centre of the base and P is a point on the circumference of the base.

Calculate

(i) VP, [2]

(ii) $V\widehat{P}O$, [2]

(iii) the circumference of the base of the cone, taking π to be 3·142. [2]

A similar cone has a vertical height of 12 cm.

(iv) Write down, in the form 1:n, the ratio of the volume of the smaller cone to that of the larger cone. [2]

4 (a)
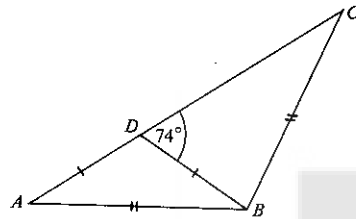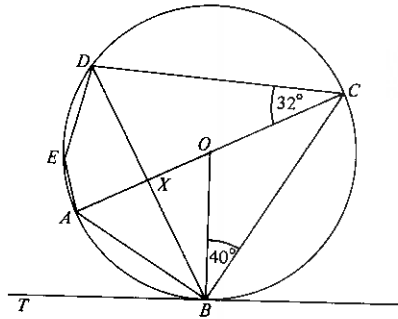

In the triangle ABC, $AB = BC$. The point D on AC is such that $AD = DB$ and $B\widehat{D}C = 74°$.

Calculate (i) $D\widehat{A}B$,

(ii) $D\widehat{B}C$.

[3]

(b)

In the diagram, O is the centre of the circle through A, B, C, D and E and TB is the tangent at B. The diameter AC and chord DB intersect at X.

Given that $O\widehat{B}C = 40°$ and $D\widehat{C}A = 32°$, calculate

(i) $A\widehat{B}T$,

(ii) $O\widehat{A}B$,

(iii) $A\widehat{E}D$,

(iv) $C\widehat{X}D$.

[5]

5 The equation of a straight line is $3y + 2x + 6 = 0$.

Calculate

(i) the gradient of the line, [1]

(ii) the coordinates of the point where the line crosses the y-axis, [2]

(iii) the coordinates of the point at which the line intersects the line $y = 4$, [2]

(iv) the equation of the line, parallel to the given line, which passes through the point (5, 2). [3]

Section B [60 *marks*]

Answer **five** *questions in this section.*

Each question in this section carries twelve marks.

6 A salesman was paid his annual salary in twelve equal monthly instalments. In addition at the end of each year, he was paid a bonus which amounted to 7% of the value of his total annual sales.

Given that his annual salary was £5100 and that his total sales during the first year amounted to £20 000, calculate

(i) his monthly salary, [1]

(ii) his total income in the first year. [2]

During the second year his annual salary remained unchanged but his total income for the year amounted to £9440.

(iii) Calculate his total sales during the second year. [4]

In the third year his annual salary was increased to £5865 and his bonus was increased to 8% of the value of his total annual sales. Calculate

(iv) the percentage increase in his annual salary, [2]

(v) the sales, correct to the nearest £1000, he had to achieve in the third year, if his total income during the year was to be at least £13 000. [3]
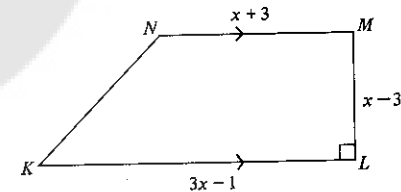
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7 The angles A, B and C of a triangle ABC are 45°, 54° and 81° respectively. BC, the shortest side of the triangle, is 12 cm long. Calculate

(i) the length of the longest side of the triangle, [4]

(ii) the length of the shortest perpendicular height of the triangle. [2]

The perpendicular bisector of BC meets BC at M and BA at N. Calculate

(iii) MN, [3]

(iv) CN. [3]

8 (a)

$KLMN$ is a trapezium in which KL is parallel to NM and $K\widehat{L}M = 90°$.

(i) Given that $KL = (3x - 1)$ cm, $NM = (x + 3)$ cm and $LM = (x - 3)$ cm, find, in terms of x, an expression for the area of the trapezium.

(ii) Given also that the area of the trapezium is 15 cm², form an equation in x and show that it reduces to
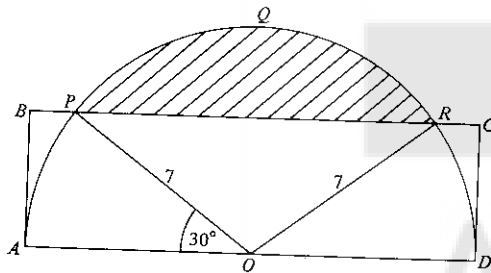
$$2x^2 - 5x - 18 = 0.$$

(iii) Solve this equation and hence find the length of LM. [8]

(b) Given that $p = 2t + 1$ and $q = t^2 - 1$,

(i) find the values of p and q when $t = -5$,

(ii) express $\dfrac{p - 4}{4q - 5}$ in terms of t, giving your answer in its simplest form. [4]
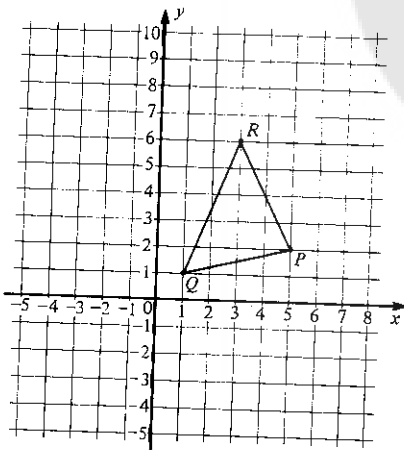

9

$ABCD$ is a rectangle and O is the mid-point of AD.

A semicircle of radius 7 cm is drawn on AD as diameter. The semicircle cuts the side BC at P and R such that $P\widehat{O}A = 30°$. Calculate

(i) AD, [1]

(ii) AB, [2]

(iii) BP. [2]

Taking π to be $\dfrac{22}{7}$, calculate

(iv) the length of the arc PQR, [3]

(v) the area of the shaded segment PQR. [4]

10

The triangle PQR with vertices $P(5, 2)$, $Q(1, 1)$ and $R(3, 6)$ is shown in the diagram.

(i) An enlargement maps $\triangle PQR$ onto $\triangle PAB$. Given that the coordinates of A are $(m, 0)$, find

(a) the centre of the enlargement,

(b) the value of m,

(c) the scale factor of the enlargement,

(d) the coordinates of the point B,

(e) the ratio of the area of $\triangle PAB$ to that of $\triangle PQR$. [5]

(ii) C is the point $(2, 5)$ and D is the point $(6, 3)$. Given that a single transformation maps $\triangle PQR$ onto $\triangle CQD$,

(a) describe the transformation fully,

(b) write down the matrix which represents this transformation. [4]

(iii) Given that $PQRS$ is a parallelogram,

(a) write down the coordinates of the point S,

(b) describe fully the single transformation which will map $\triangle PQR$ onto $\triangle RSP$. [3]

11 (a)
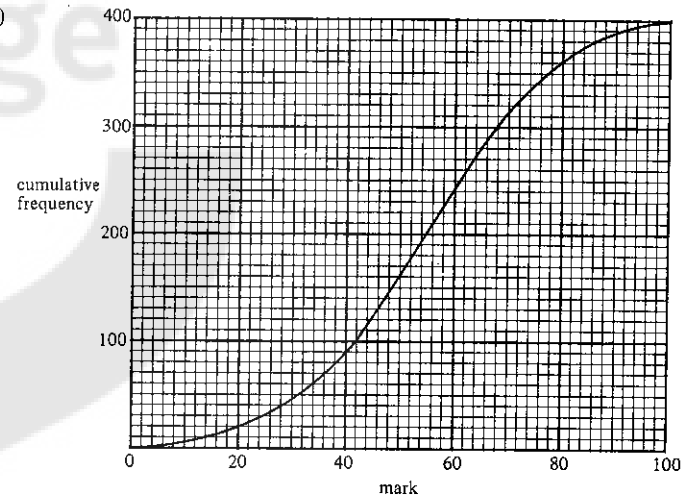
The diagram is the cumulative frequency curve for the marks of 400 candidates in an examination. Use the curve to estimate, as accurately as possible,

(i) the median mark,

(ii) the inter-quartile range,

(iii) the pass mark, given that 70% of the candidates passed the examination,

(iv) the probability that a candidate scored 80% or less. [7]

(b) A bag contains four counters, one marked with the letter *A*, one with the letter *B* and two with the letter *L*.

The counters are drawn at random from the bag, one at a time, without replacement.

In each of the following cases calculate the probability that

(i) the first two counters to be drawn out will each have the letter *L* marked on them,

(ii) the second counter to be drawn out will be that with the letter *B* marked on it,

(iii) the order in which the counters are drawn will spell out the word *B A L L*.   [5]

12   Answer the whole of this question on a sheet of plain paper.

(a)   (i) Construct a triangle *ABC* in which *AB* = 9 cm, *BC* = 7 cm and *ABC* = 38°. Measure, and write down, the length of *AC*.

(ii) Find, and mark clearly with the letter *P*, the two points which are 6 cm from *B* and equidistant from *AC* and *AB*.

(iii) The point *Q*, which lies inside the triangle *ABC*, is such that its distance from *B* is less than 6 cm and it is nearer to *AC* than to *AB*.

Indicate clearly, by shading, the region in which *Q* lies.   [7]

(b)
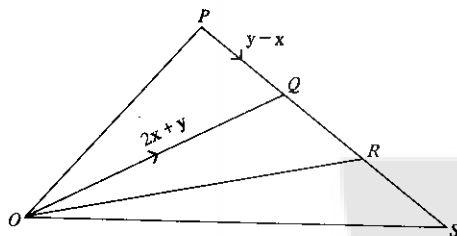
In the diagram, *PQRS* is a straight line and *PQ* = *QR* = *RS*.

Given that $\overrightarrow{OQ} = 2\mathbf{x} + \mathbf{y}$ and $\overrightarrow{PQ} = \mathbf{y} - \mathbf{x}$, express, as simply as possible, in terms of **x** and/or **y**,

(i) $\overrightarrow{RQ}$,

(ii) $\overrightarrow{OP}$,

(iii) $\overrightarrow{OS}$.   [5]

13   Answer the whole of this question on a sheet of graph paper.

A farmer who intended to keep sheep and cows on his farm asked each of his four sons how many sheep and/or cows he should keep.

Alan suggested that there should be more than 10 cows.

Brian suggested that the number of sheep should be at least 20 but not more than 50.

Charles suggested that the total number of sheep and cows should be less than 70.

David suggested that the number of sheep should be greater than or equal to the number of cows.

Taking *s* to be the number of sheep and *c* to be the number of cows, write down the inequalities which represent these conditions.   [4]

The point (*s*, *c*) represents *s* sheep and *c* cows. Using a scale of 2 cm to represent 10 sheep on the horizontal axis and a scale of 2 cm to represent 10 cows on the vertical axis construct, and indicate clearly by shading the **unwanted** regions, the region in which (*s*, *c*) must lie.   [6]

Assuming the farmer took all his sons' suggestions into account and that, when he came to sell the animals, he made a profit of £50 on each sheep and £100 on each cow, find the minimum number of cows he kept on his farm to ensure a profit of at least £4 000.   [2]